



DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

COURSE MATERIALS



EC 403: MICROWAVE & RADAR ENGINEERING

VISION OF THE INSTITUTION

To mould true citizens who are millennium leaders and catalysts of change through excellence in education.

MISSION OF THE INSTITUTION

NCERC is committed to transform itself into a center of excellence in Learning and Research in Engineering and Frontier Technology and to impart quality education to mould technically competent citizens with moral integrity, social commitment and ethical values.

We intend to facilitate our students to assimilate the latest technological know-how and to imbibe discipline, culture and spiritually, and to mould them in to technological giants, dedicated research scientists and intellectual leaders of the country who can spread the beams of light and happiness among the poor and the underprivileged.

ABOUT DEPARTMENT

- ◆ Established in: 2002
- ◆ Course offered : B.Tech in Electronics and Communication Engineering
M.Tech in VLSI
- ◆ Approved by AICTE New Delhi and Accredited by NAAC
- ◆ Affiliated to the University of Dr. A P J Abdul Kalam Technological University.

DEPARTMENT VISION

Providing Universal Communicative Electronics Engineers with corporate and social relevance towards sustainable developments through quality education.

DEPARTMENT MISSION

- 1) Imparting Quality education by providing excellent teaching, learning environment.
- 2) Transforming and adopting students in this knowledgeable era, where the electronic gadgets (things) are getting obsolete in short span.
- 3) To initiate multi-disciplinary activities to students at earliest and apply in their respective fields of interest later.
- 4) Promoting leading edge Research & Development through collaboration with academia & industry.

PROGRAMME EDUCATIONAL OBJECTIVES

PEOI. To prepare students to excel in postgraduate programmes or to succeed in industry / technical profession through global, rigorous education and prepare the students to practice and innovate recent fields in the specified program/ industry environment.

PEO2. To provide students with a solid foundation in mathematical, Scientific and engineering fundamentals required to solve engineering problems and to have strong practical knowledge required to design and test the system.

PEO3. To train students with good scientific and engineering breadth so as to comprehend, analyze, design, and create novel products and solutions for the real life problems.

PEO4. To provide student with an academic environment aware of excellence, effective communication skills, leadership, multidisciplinary approach, written ethical codes and the life-long learning needed for a successful professional career.

PROGRAM OUTCOMES (POS)

Engineering Graduates will be able to:

1. **Engineering knowledge:** Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.
2. **Problem analysis:** Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.
3. **Design/development of solutions:** Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.
4. **Conduct investigations of complex problems:** Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.
5. **Modern tool usage:** Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations.
6. **The engineer and society:** Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.
7. **Environment and sustainability:** Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable

development.

8. **Ethics:** Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.
9. **Individual and team work:** Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.
10. **Communication:** Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.
11. **Project management and finance:** Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.
12. **Life-long learning:** Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

PROGRAM SPECIFIC OUTCOMES (PSO)

PSO1: Ability to Formulate and Simulate Innovative Ideas to provide software solutions for Real-time Problems and to investigate for its future scope.

PSO2: Ability to learn and apply various methodologies for facilitating development of high quality System Software Tools and Efficient Web Design Models with a focus on performance optimization.

PSO3: Ability to inculcate the Knowledge for developing Codes and integrating hardware/software products in the domains of Big Data Analytics, Web Applications and Mobile Apps to create innovative career path and for the socially relevant issues.

COURSE OUTCOMES

EC 403

SUBJECT CODE: EC 403	
COURSE OUTCOMES	
C403.1	To introduce the various microwave sources, their principle of operation and measurement of various parameters
C403.2	To study and analyze the working principles of Klystron and magnetron oscillators.
C403.3	To study and analyze the working principles of Travelling wave tube and its microwave measurement.
C403.4	To study the various microwave hybrid circuits and formulate their S matrices
C403.5	Analyze tunnel diode gunn diode and its working principles
C403.6	To understand the basic concepts, types, working of radar and introduce to radar transmitters

MAPPING OF COURSE OUTCOMES WITH PROGRAM OUTCOMES

CO'S	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
C403.1	3	3										1
C403.2	3	3	2	2								
C403.3	3	3	3	2							2	
C403.4	3	3	3	2	1						1	
C403.5	3	3	2	2	1						1	
C403.6	3	3	2				2					1
C403	3	2	2	2	2		2				2	1

CO'S	PSO1	PSO2	PSO3
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C403.1		1	1
C403.2	2	3	
C403.3	2	3	
C403.4	2	3	
C403.5	3		
C403.6			1
C403	2	3	1

SYLLABUS

COURSE CODE	COURSE NAME	L-T-P-C	YEAR OF INTRODUCTION
EC403	MICROWAVE & RADAR ENGINEERING	3-0-0-3	2016
Prerequisite: EC303 Applied Electromagnetic Theory, EC306 Antenna & Wave Propagation			
Course objectives:			
<ul style="list-style-type: none">To introduce the various microwave sources, their principle of operation and measurement of various parametersTo study the various microwave hybrid circuits and formulate their S matrices.To understand the basic concepts, types, working of radar and introduce to radar transmitters and receivers.			
Syllabus:			
Microwaves: introduction, advantages, Cavity Resonators, Microwave vacuum type amplifiers and sources, Klystron Amplifiers, Reflex Klystron Oscillators, Magnetron oscillators, Travelling Wave Tube, Microwave measurements, Microwave hybrid circuits, Directional couplers, Solid state microwave devices, Gunn diodes, Radar, MTI Radar, Radar Transmitters, Radar receivers.			
Expected outcome:			
The students will be able to understand the basics of microwave engineering and radar systems.			
Text Books:			
<ul style="list-style-type: none">Merrill I. Skolnik, Introduction to Radar Systems, 3/e, Tata McGraw Hill, 2008.Samuel Y. Liao, Microwave Devices and Circuits, 3/e, Pearson Education, 2003.			
References:			
<ul style="list-style-type: none">Das, Microwave Engineering, 3/e, McGraw Hill Education India Education, 2014Kulkarni M, Microwave and Radar Engineering, 4/e, Umesh Publications, 2012.Rao, Microwave Engineering, 2/e, PHI, 2012.Robert E. Collin, Foundation of Microwave Engineering, 2/e, Wiley India, 2012.			
Course Plan			
Module	Course contents	Hours	End Sem. Exam Marks
I	Microwaves: introduction, advantages, Cavity Resonators - Rectangular and Circular wave guide resonators- Derivation of resonance frequency of Rectangular cavity.	4	15%
	Microwave vacuum type amplifiers and sources: Klystron Amplifiers - Re-entrant cavities, Velocity modulation, Bunching (including analysis), Output power and beam	4	
II	Reflex Klystron Oscillators: Derivation of Power output, efficiency and admittance	2	15%
	Magnetron oscillators: Cylindrical magnetron, Cyclotron angular frequency, Power output and efficiency.	3	
FIRST INTERNAL EXAM			
III	Travelling Wave Tube: Slow wave structures, Helix TWT, Amplification process, Derivation of convection current, axial electric field, wave modes and gain.	4	15%
	Microwave measurements: Measurement of impedance, frequency and power	2	

IV	Microwave hybrid circuits: Scattering parameters, Waveguide tees- Magic tees, Hybrid rings, Corners, Bends, and Twists. Formulation of S-matrix.	5	15%
	Directional couplers: Two hole directional couplers, S-matrix of a directional coupler. Circulators and isolators.	4	
SECOND INTERNAL EXAM			
V	Solid state microwave devices: Microwave bipolar transistors, Physical structures, Power frequency limitations equivalent circuit. Principle of Tunnel diodes and tunnel	4	20%
	Gunn diodes: Different modes, Principle of operation Gunn Diode Oscillators.	2	
VI	Radar: The simple Radar equation. Pulse Radar, CW Radar, CW Radar with non zero IF, Equation for doppler frequency FM-CW Radar using sideband super heterodyne receiver. MTI Radar -Delay line canceller, MTI Radar with power amplifier & power oscillator, Non coherent MTI Radar, Pulse	5	20%
	Radar Transmitters: Radar Modulator-Block diagram, Radar receivers - noise figure, low noise front ends, Mixers, Radar Displays	3	

QUESTION BANK

MODULE I				
Q:NO:	QUESTIONS	CO	KL	PAGE NO:
1	Develop and Derive the equation for resonant frequency for a rectangular cavity resonator.	CO1	K3	4
2	Write and formulate the equations necessary for resonant frequency operation in a rectangular cavity resonators. Also give the f_r relating to circular cavity resonators.	CO1	K3	6
3	Mention the significance of re-entrant cavities also list its types.	CO1	K2	8
4	Interpret and obtain an equation of ' f_r ' for a rectangular cavity resonator.	CO1	K3	10
5	Sketch and brief the Applegate diagram for 2-cavity Klystron amplifier.	CO1	K3	12
6	Prove that a coaxial reentrant cavity support infinite number of resonant frequencies.	CO1	K5	16
7	Give the formula relating to Q factor of cavity resonator.	CO1	K2	18
8	Indicate the significance re-entrant cavities in microwave tubes also list the different types of re-entrant cavities.	CO1	K2	20
9	Analyze the bunching process in a two cavity klystron amplifier and derive the bunching parameter.	CO1	K4	23
10	Develop, Define and derive velocity modulation in two cavity klystron amplifier	CO1	K3	30
11	Identify, how velocity modulation changes to current density modulation in klystron amplifier.	CO1	K3	30
12	Determine the resonant frequency of an air filled rectangular cavity operating in the dominant mode with dimensions as $a=4\text{cm}$, $b=5\text{cm}$ and $d=6\text{cm}$.	CO1	K5	31
13	Carry out and find the resonant frequency of the first 5 lowest modes of an air filled rectangular cavity of dimensions $5\text{cm} * 4\text{cm} * 2.5\text{cm}$ also identify the dominant mode among them.	CO1	K3	33
14	Given the parameters of a two cavity klystron amplifier beam voltage $V_0=1000\text{V}$, $R_0=40\text{K}\Omega$,	CO1	K5	35

	$R_{sh}=30K\Omega$, beam current=50mA, operating frequency $f=10GHz$. Gap spacing =1mm, spacing between the two cavities $L=5cm$, Determine: (a) The input gap voltage to give the maximum voltage V_1 (b) The Voltage gain.(A_v) (c) The efficiency of the amplifier.			
15	A two cavity klystron amplifier has the following parameters $V_o=1000V$, $R_o=40K\Omega$, $I_o=25mA$, $f=3GHz$. Gap spacing in either cavity $d=1mm$, spacing between the two cavities $L=4cm$, shunt impedance $R_{sh}= 30K\Omega$. Solve : a) The input gap voltage to give the maximum voltage V_1 . b) The Voltage gain.(A_v) c) The efficiency of the amplifier.			
MODULE II				
1	Sketch and brief the Applegate diagram for reflex Klystron	CO2	K3	49
2	How oscillation generated in reflex klystron?	CO2	K2	51
3	Derive the velocity modulation equation for reflex Klystron.	CO2	K2	52
4	Give the performance characteristics and applications of reflex klystron.			
4	How oscillation generated in Magnetron?	CO2	K2	53
5	Examine with the help of neat sketches and sufficient equations, the working of a cylindrical magnetron.	CO2	K4	54
	Analyze and sketch the structure of 8 cavity magnetron and explain its bunching process	CO2	K4	58
6	Analyze and sketch the Applegate diagram for $2\frac{3}{4}$ mode in reflex klystron	CO2	K4	62
7	Interpret and obtain the equation for power and efficiency of reflex klystron.	CO2	K3	63
7	A reflex klystron is operated at 9GHz with dc beam voltage 600V for $1\frac{3}{4}$ mode, repeller space length 1mm, and dc beam current 10mA. The beam coupling co-efficient is assumed to be one. Solve and Calculate : a. The value of the repeller voltage V_r .	CO2	K3	65

	b. Output power. c. electronic efficiency.			
8	Examine the electronic admittance of the gap in the case of reflex klystron. With admittance diagram explain the condition required for oscillation in reflex klystron	CO2	K4	66
9	Develop and derive the velocity modulation equation for reflex Klystron	CO2	K3	68
10	Sketch the equivalent circuit of reflex klystron.	CO2	K3	70
11	Compare and contrast Klystron with Magnetron.	CO2	K2	72
MODULE III				
1	Describe on various types of slow wave structures.	CO3	K2	85
2	Sketch and brief on Helix TWT slow wave structure.	CO3	K3	87
3	Indicate the purpose of slow wave structures? List its types.	CO3	K2	89
4	Explain the operating principle of Helix TWT.	CO3	K2	90
5	Develop and derive hull cut-off voltage equation.	CO3	K3	91
6	Discuss the basic operation of travelling wave tube and give the significance of its slow wave structure	CO3	K2	94
7	Conclude and derive the necessary equations relating to amplification process in TWT	CO3	K4	95
8	Prove that the axial electric field of TWT varies with convection current.	CO3	K4	96
9	Develop and derive the expression for the axial electrical field in the TWT.	CO3	K2	97
10	Inspect with neat diagram any two methods to measure impedance at microwave frequencies.	CO3	K4	98
11	Sketch the block diagram of a typical microwave bench setup and label all the parts.	CO3	K3	100
MODULE IV				
1	Paraphrase and define the S matrix of a two port network. Represent the logical variables used mathematically and with the aid of a figure.	CO4	K2	105
2	Determine the coupling, directivity and isolation (in dBs) of a lossless directional coupler carrying the following: Incident	CO4	K5	

	power: 40mW, power at the coupling port: 10mW, and power at the decoupled port: 0.1mW.			
3	Rephrase and discuss the operation of E-Plane tee.	CO4	K2	103
4	Rephrase and discuss the operation of H-Plane tee	CO4	K2	105
5	Develop and derive the S-matrix for E-Plane tee.	CO4	K3	107
6	Develop and Derive the S-matrix for H-Plane tee	CO4	K3	110
7	Inspect a two hole directional coupler and derive its S matrix.	CO4	K4	111
8	Examine the constructional features of magic tees and derive its S Matrix. Why are they called so?	CO4	K4	112
9	Dissect with schematic describe the operation of a three port circulator. Obtain the simplified S matrix of a perfectly matched, lossless three port circulator.	CO4	K4	112
10	Debate and derive the expression of scattering matrix for directional coupler.	CO4	K4	113
11	Experiment the constructional features of two hole directional coupler and derive S matrix.	CO4	K4	114
12	Investigate how isolators can support only forward direction waves.	CO4	K4	115

MODULE V

1	Distinguish between microwave transistors and TEDs.	CO5	K2	117
2	Sketch and brief the different geometries of microwave power transistor?	CO5	K3	119
3	Discuss and brief on tunnel diode oscillator.	CO5	K2	120
4	Give the advantages and disadvantages of Gunn diode.	CO5	K2	122
5	Explain series and parallel loading in tunnel diode.	CO5	K2	123
6	A certain silicon microwave transistor has the following parameters: Reactance = 1Ω , Transit-time cut off frequency = 4 GHz, Maximum electric field = 1.6×10^5 V/cm, Saturation drift velocity = 4×10^5 cm/s. Determine the maximum power that the transistor can carry.	CO5	K5	124

7	Analyze in detail the principle of a GUNN diode. Draw the I V characteristics	C05	K4	126
8	Sketch the energy band diagrams of tunnel diode. Interpret the operation of tunnel diode with the help of I-V characteristics.	C05	K3	128
9	Debate the Ridley -Watkins -Hilsum theory and derive the condition for negative resistance.	C05	K4	129
MODULE VI				
1	Explain the basic principles of radar system.	C06	K2	130
2	Investigate with neat diagram, the working of CW radar with non-zero IF.	C06	K4	130
3	Develop and derive RADAR range equation.	C06	K3	131
4	Explain the more commonly used radar displays.	C06	K2	132
5	Interpret how the noise figure of a radar receiver is monitored.	C06	K2	133
6	Debate and derive the minimum detectable signal of a RADAR.	C06	K4	134
7	Infer on low noise front ends? Describe in detail the utility of low noise front.	C06	K2	135
8	Review the Doppler effect and derive the equation for Doppler frequency.	C06	K4	136
9	Prove that the product of the maximum unambiguous range R_{un} and the first blind speed v_1 is equal to $c \lambda/4$.	C06	K5	137
10	A guided missile tracking radar has the following specifications Transmitted Power = 400 kW ; Pulse repetition frequency = 1500 pps ; Pulse width = 0.8 μ sec Determine Unambiguous range, Duty cycle, Average power and suitable bandwidth of the radar.	C06	K5	138

MODULE - I

Introduction to Microwaves

Microwave is the region in electromagnetic spectrum in the range between 3 - 300 GHz. The wavelength of microwave range between 30 - 3 cm.

Microwave engineering is an important consideration in the development of high resolution radar, Communication channel etc.

Microwave Frequency Bands

Band	Frequency (GHz)
VHL	0.5-1
L	1-2
S	2-4
C	4-8
X	8-12.4
K _u	12.4-18
K	18-26.5
K _a	26.5-40
V	40-75
W	75-110
D	110-170

Advantages of Microwaves.

- High operating frequency i.e. they can carry large amount of information.

- Short wavelength. Hence, short antennas are used.
- Easily pass through ionosphere.
- Can be used for space & satellite communication.
- Few repeaters are needed.
- Minimum cross talk.
- Highly reliable.
- Increased Bandwidth.
- High directivity.

Disadvantages of Microwave.

- Difficult to analyse and design.
- Difficult to implement circuit components.
- Transmission time is high.

Applications of Microwave.

- Telecommunication.
- Radar.
- Radio broadcasting.
- Microwave Ovens.
- Astronomy.
- Remote Sensing.
- Remote Radiometry.
- Military Application etc.

NOTES
the learning companion

Microwave Cavity Resonators.

A cavity resonator is a metallic enclosure that confines electromagnetic energy. The stored electric and magnetic energy inside the cavity determine inductance and capacitance. The energy dissipated by the conductivity of the walls of the cavity determines the resistance. The parameters which describe the performance of a resonator are

i) Resonant frequency: It is the frequency at which the energy in the cavity attains a max value of $2W_e$ or $2W_m$
where W_e - energy stored in electric field.
 W_m - Energy stored in magnetic field.

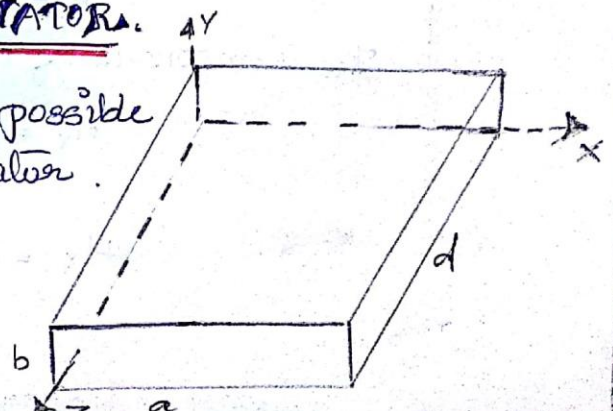
ii) Quality Factor, Q : It is the measure of frequency selectivity of the cavity
$$Q = 2\pi \times \frac{\text{max energy stored}}{\text{Energy dissipated per cycle}}$$

iii) Modes of the Cavity: A given resonator has infinite number of modes and each mode corresponds to a definite frequency. When the freq of the signal is equal to resonant freq, a max amplitude of wave occurs and the energies stored in electric and magnetic fields are equal. The mode having lowest resonant frequency is known as dominant mode.

RECTANGULAR CAVITY RESONATOR.

There are 2 modes of propagation possible inside a rectangular cavity resonator.

They are TE mode in which electric mode is transverse and TM mode in which magnetic mode is transverse.



for TE mode $E_z = 0$ and solution may be derived from H_z component

For TM mode $H_z = 0$ and solⁿ is deriv^d from E_z component

TE Mode

The H_z Component is defined by the equation.

$$H_z = H_0 \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) \sin\left(\frac{p\pi z}{d}\right)$$

where H_0 - amp of magnetic field

m - no: of waves in x direction

n - no: of waves in y direction

p - no: of waves in z direction

The Component H_y is defined as $\frac{1}{k_c^2} \frac{\partial^2 H_z}{\partial y \partial z}$

$$H_y = \frac{1}{k_c^2} \frac{\partial^2 H_z}{\partial y \partial z}$$

k_c - cut off value.

$$H_y = \frac{1}{k_c^2} \frac{\partial}{\partial y} \left(\frac{\partial H_z}{\partial z} \right)$$

$$= \frac{1}{k_c^2} \frac{\partial}{\partial y} \left[H_0 \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{b} \cos\left(\frac{p\pi z}{d}\right) \right] \frac{p\pi}{d}$$

$$= \frac{1}{k_c^2} H_0 \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \left(-\frac{n\pi}{b}\right) \cos \frac{p\pi z}{d} \left(\frac{p\pi}{d}\right)$$

$$= -\frac{H_0}{k_c^2} \left[\cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \left(\frac{n\pi}{b}\right) \cos\left(\frac{p\pi z}{d}\right) \left(\frac{p\pi}{d}\right) \right]$$

The Component H_x is defined as

$$H_x = \frac{1}{k_c^2} \frac{\partial^2 H_z}{\partial x \partial z}$$

$$H_x = -\frac{H_0}{k_c^2} \left[\sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) \cos\left(\frac{p\pi z}{d}\right) \left(\frac{m\pi}{a}\right) \left(\frac{p\pi}{d}\right) \right]$$

for TE mode $E_z = 0$, the Component E_y is defined by

$$E_y = \frac{j\omega\mu H_0}{k_c^2} \frac{\partial H_z}{\partial x}$$

$$E_y = \frac{j\omega\mu H_0}{k_c^2} \frac{-m\pi}{a} H_0 \left[\sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) \sin\left(\frac{p\pi z}{d}\right) \right]$$

The Component E_x is defined by

$$E_x = -\frac{j\omega\mu H_0}{k_c^2} \frac{\partial H_z}{\partial y}$$

$$= +\frac{j\omega\mu H_0}{k_c^2} \frac{n\pi}{b} H_0 \left[\cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \sin\left(\frac{p\pi z}{d}\right) \right]$$

TM Mode

The E_z Component is defined by the equation

$$E_z = E_0 \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \cos \frac{p\pi z}{d}$$

$$E_y = \frac{E_0}{k_c^2} \frac{\partial^2 E_z}{\partial y \partial z}$$

$$E_x = \frac{E_0}{k_c^2} \frac{\partial^2 E_z}{\partial x \partial z}$$

$$H_z = 0$$

$$H_y = -\frac{j\omega\epsilon E_0}{k_c^2} \frac{\partial E_z}{\partial x}$$

$$H_x = \frac{j\omega\epsilon E_0}{k_c^2} \frac{\partial E_z}{\partial y}$$

We have,

$$E_y = \frac{E_0}{k_c^2} \frac{\partial^2 E_z}{\partial y \partial z}$$

$$= -\frac{E_0}{k_c^2} \left(\frac{n\pi}{b}\right) E_0 \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) \cos p\left(\frac{p\pi}{d}\right) \sin\left(\frac{p\pi z}{d}\right)$$

$$= -\frac{E_0^2}{k_c^2} \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) \sin\left(\frac{p\pi z}{d}\right) \left(\frac{n\pi}{b}\right) \left(\frac{p\pi}{d}\right)$$

$$E_x = \frac{E_0}{k_c^2} \frac{\partial^2 E_z}{\partial x \partial z}$$

$$= \frac{-E_0^2}{k_c^2} \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \sin\left(\frac{p\pi z}{d}\right) \left(\frac{m\pi}{a}\right) \left(\frac{p\pi}{d}\right)$$

$$H_z = 0.$$

The Component H_y is defined as

$$H_y = -\frac{j\omega\epsilon}{k_c^2} E_0 \frac{\partial E_z}{\partial x}$$

$$= -\frac{j\omega\epsilon}{k_c^2} E_0^2 \left(\frac{m\pi}{a}\right) \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \cos\left(\frac{p\pi z}{d}\right)$$

$$H_x = \frac{j\omega\epsilon}{k_c^2} E_0^2 \left(\frac{n\pi}{b}\right) \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) \cos\left(\frac{p\pi z}{d}\right)$$

CIRCULAR CAVITY RESONATORS.

A circular cavity resonator is a circular waveguide with two ends closed by a metallic wall. The field components inside the cavity are described as TE_{nmp} and TM_{nmp} .

TE MODE

It is described by the equation,

$$H_z = H_0 J_n(x'_{nmp}/a) \cos n\phi \sin\left(\frac{p\pi z}{d}\right)$$

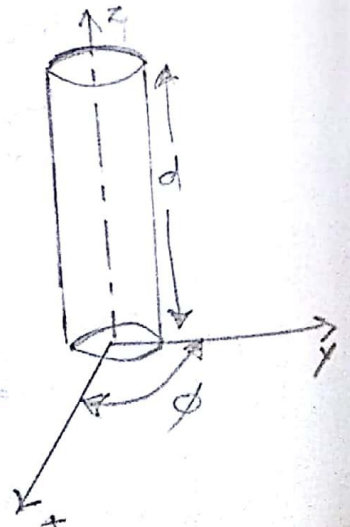
where a , p and ϕ are the cylindrical coordinates.

TM MODE

It is defined by the equation:

$$E_z = E_0 J_n(x'_{nmp}/a) \cos n\phi \sin\left(\frac{p\pi z}{d}\right)$$

For the rectangular cavity resonator the resonant frequency is given by



$$f_r = \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2}$$

for circular cavity resonator, the resonant frequency is given by

$$f_r = \frac{1}{2\pi\sqrt{\mu\epsilon}} \sqrt{\left(\frac{x_{nmp}}{a}\right)^2 + \left(\frac{p\pi}{d}\right)^2}$$

KLYSTRON AMPLIFIERS.

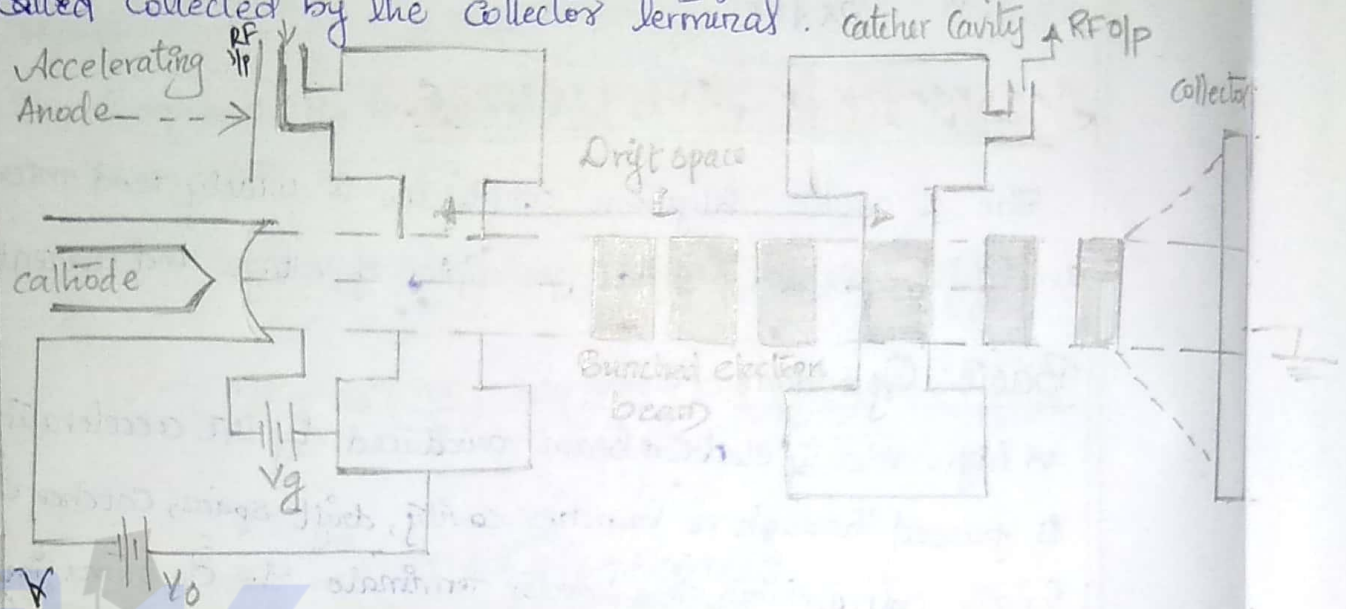
The 2 cavity klystron amplifier is widely used microwave amplifier, operated by the principles of voltage and current modulation.

Basic Operations

A high velocity electron beam produced by the accelerating anode is passed through a buncher cavity, drift space, catcher cavity & finally collected by the collector terminals. The electrons injected from the cathode is accelerated by applying a DC voltage " V_0 ".

They arrive at the first cavity that is the buncher cavity or input cavity with uniform velocity. At the buncher cavity there electrons encounter signal voltage or gap voltage. The electrons that pass through the zero of the gap voltage pass with unchange velocity. The electrons that pass through positive half cycles of the gap voltage undergo acceleration in velocity. The electrons that pass through negative half cycles of the gap voltage undergo retardation in velocity. [As a result of these the electrons get bunched together as they travelled through the drift space]. The variation in electron velocity in drift space is called velocity modulation. [The buncher cavity velocity modulates the electron beam]. This electron beam induces a RF current in this field is opposite to

the i/p cavity. Thus the kinetic energy is transferred from the electrons to the field capture cavity. The second cavity is called capture cavity since it captures energy from the bunch electron beam. The electrons emerging from the capture cavity are collected by the collector terminal.



Velocity Modulation

The velocity of electrons before entering the buncher cavity is given by

$$V_0 = \sqrt{\frac{2eV_0}{m}}$$

where m is the mass of the electron

e - charge of electron

V_0 is the cathode potential.

On substituting the values of e and m the equation reduces to

$$V_0 = 0.596 \times 10^6 \sqrt{V_0} \text{ m/sec} \quad \text{--- (1)}$$

When the microwave signal is applied to the i/p terminal the gap voltage is given by

$$V_0 = V_1 \sin \omega t \quad \text{--- (2)}$$

V_1 is the amplitude of the signal.

The average transit time through the gap at distance d ,

$$\tau = \frac{d}{v_0} = t_1 - t_0 \quad (3)$$

where t_0 is the time at which beam reaches the buncher cavity.

t_1 - time at which the beam leaves the buncher cavity.

The average transit angle $\theta_g = \omega \tau = \omega \left(\frac{d}{v_0} \right) = \omega(t_1 - t_0) \quad (4)$

The average microwave voltage in the buncher cavity is

$$\begin{aligned} \langle V_s \rangle &= \frac{1}{\tau} \int_{t_0}^{t_1} V_1 \sin \omega t \, dt \\ &= \frac{1}{\tau} - V_1 \left(\frac{\cos \omega t}{\omega} \right)_{t_0}^{t_1} = -\frac{V_1}{\tau \omega} (\cos \omega t)_{t_0}^{t_1} \\ &= -\frac{V_1}{\tau \omega} (\cos \omega t_1 - \cos \omega t_0) \\ &= \frac{V_1}{\tau \omega} (\cos \omega t_0 - \cos \omega t_1) \quad (5) \end{aligned}$$

From eq (4) $\frac{\omega d}{v_0} = \omega(t_1 - t_0)$

$$\omega t_1 = \frac{\omega d}{v_0} + \omega t_0 \quad (6)$$

$$\omega t_1 = \omega \left(\frac{d}{v_0} + t_0 \right)$$

Sub. eq (6) in eqn (5)

$$\langle V_s \rangle = \frac{V_1}{\tau \omega} \left[\cos \omega t_0 - \cos \left(\frac{\omega d}{v_0} + \omega t_0 \right) \right] \quad (7)$$

let $\omega t_0 + \frac{\omega d}{2 v_0} = \omega t_0 + \frac{\theta_g}{2} = A,$

$$\frac{\omega d}{2 v_0} = \frac{\theta_g}{2} = B.$$

$$\begin{aligned}
 A+B &\Rightarrow \omega t_0 + \frac{\phi_g}{2} + \frac{\phi_g}{2} \\
 &= \omega t_0 + \frac{2\phi_g}{2} \\
 &= \omega t_0 + \phi_g
 \end{aligned}$$

$$\begin{aligned}
 A-B &\Rightarrow \omega t_0 + \frac{\phi_g}{2} - \frac{\phi_g}{2} \\
 &= \omega t_0
 \end{aligned}$$

$$\begin{aligned}
 A+B &= \omega t_0 + \phi_g \\
 A-B &= \omega t_0
 \end{aligned}
 \quad \left. \vphantom{\begin{aligned} A+B &= \omega t_0 + \phi_g \\ A-B &= \omega t_0 \end{aligned}} \right\} \text{--- (8)}$$

Subs. eq (8) in eq (7)

$$\langle V_s \rangle = \frac{V_1}{2\omega} [\cos(A-B) - \cos(A+B)]$$

$$= \frac{2V_1}{2\omega} \sin A \sin B$$

$$= \frac{2V_1}{2\omega} \sin\left(\omega t_0 + \frac{\phi_g}{2}\right) \sin\left(\frac{\phi_g}{2}\right)$$

$$\text{let } \omega = \phi_g$$

$$\therefore \langle V_s \rangle = \frac{2V_1 \sin(\omega t_0 + \phi_g/2) \sin(\phi_g/2)}{\phi_g}$$

$$\therefore \langle V_s \rangle = V_1 \rho_f \sin(\omega t_0 + \phi_g/2)$$

$$\boxed{\rho_f = \frac{\sin(\phi_g/2)}{(\phi_g/2)}}$$

thus

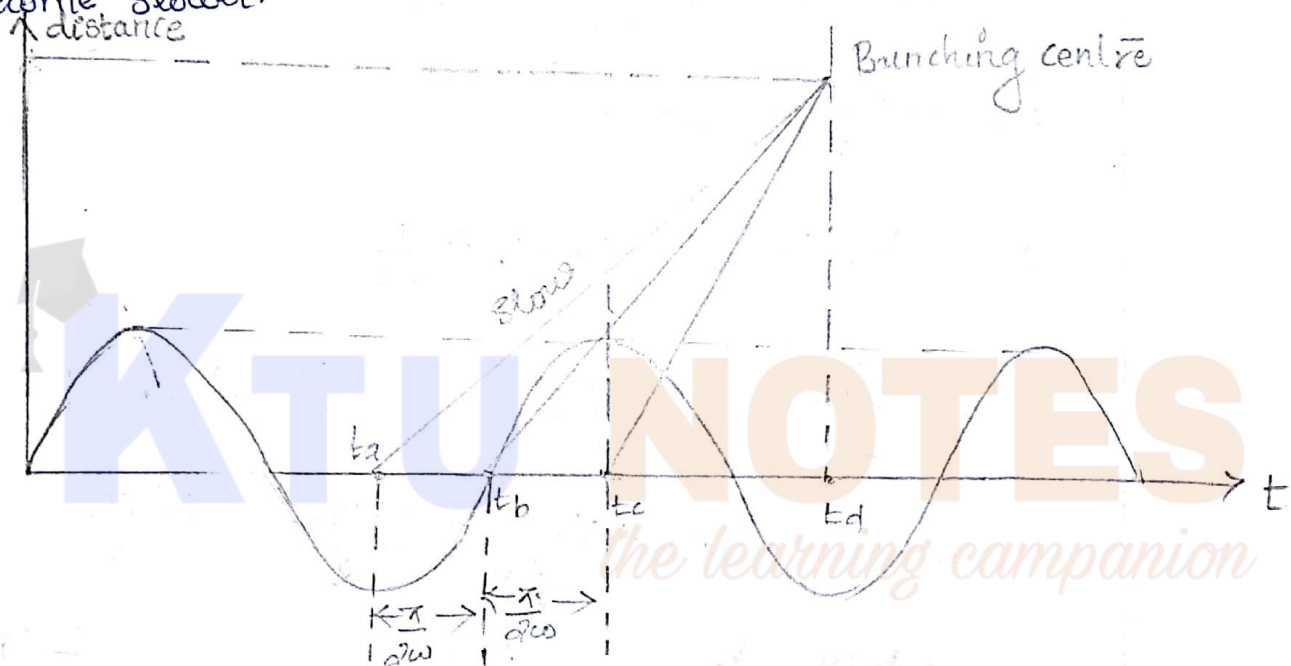
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Write a note on reentrant cavity

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Bunching Process

Once the electrons leave the buncher cavity, they drift with a velocity along the space betⁿ two cavities. The effect of velocity modulation produces, bunching of electron beam or current modulation. The electrons that pass the buncher cavity with zero voltage travel with unchanged velocity and becomes the bunching centre. Electrons that pass the bunching cavity during +ve half cycles of microwave i/p become faster and electrons that pass during the -ve half cycle become slower.



t_a - time at which maximum retardation occur.

t_b - time at which electrons have uniform velocity.

t_c - time at which maximum acceleration occur.

Bunching centre is the point at which electron density is maximum. The distance to the bunching centre

$$\Delta L = V_0 (t_d - t_b) \quad \text{--- (1)}$$

$$t_c = t_b + \frac{\pi}{2\omega}$$

$$t_b = t_a + \frac{\pi}{2\omega}$$

$$t_a = t_b - \frac{\pi}{2\omega}$$

} --- (2)

The distance of electron at t_a

$$\Delta t = V_{\min} (t_d - t_a) \\ = V_{\min} \left[t_d - t_b + \frac{\pi}{2\omega} \right] \quad - (3)$$

The distance of electron at t_c

$$\Delta t = V_{\max} (t_d - t_c) \\ = V_{\max} \left[t_d - t_b - \frac{\pi}{2\omega} \right] \quad - (4)$$

$$\text{Let } V_{\min} = V_0 \left[1 - \frac{\beta i V_p}{2V_0} \right] \quad - (5)$$

$$V_{\max} = V_0 \left[1 + \frac{\beta i V_p}{2V_0} \right] \quad - (6)$$

Sub. Eqⁿ (5) in eqⁿ (3).

$$\Delta t = V_0 \left[1 - \frac{\beta i V_p}{2V_0} \right] \left(t_d - t_b + \frac{\pi}{2\omega} \right) \\ = \left[V_0 - \frac{\beta i V_p V_0}{2V_0} \right] \left(t_d - t_b + \frac{\pi}{2\omega} \right) \\ = V_0 t_d - V_0 t_b + V_0 \frac{\pi}{2\omega} - \frac{t_d \beta i V_p}{2} + \frac{\beta i V_p t_b}{2} - \frac{\beta i V_p \pi}{4\omega} \\ = V_0 t_d - V_0 t_b + V_0 \left[\frac{\pi}{2\omega} - \frac{\beta i V_p \pi}{4\omega} \right] \quad L(7)$$

Sub eqⁿ (6) in (4)

$$\Delta t = V_{\max} V_0 \left[1 + \frac{\beta i V_p}{2V_0} \right] \left[t_d - t_b - \frac{\pi}{2\omega} \right] \\ = \left[V_0 + \frac{\beta i V_p}{2} \right] \left[t_d - t_b - \frac{\pi}{2\omega} \right] \\ = V_0 t_d - V_0 t_b - \frac{V_0 \pi}{2\omega} + \frac{\beta i V_p t_d}{2} - \frac{\beta i V_p t_b}{2} - \frac{\beta i V_p \pi}{4\omega} \quad L(8)$$

The necessary Condition at which electrons meet at a distance ΔL is

Equating eq.(7) and eq.(8)

$$\begin{aligned} \cancel{V_0 t_d} - \cancel{V_0 t_b} + \frac{V_0 \pi}{2\omega} - \frac{t_d \beta_i V_i^0}{2} + \frac{\beta_i V_i^0 t_b}{2} - \frac{\cancel{\beta_i V_i^0 \pi}}{4\omega} \\ = \cancel{V_0 t_d} - \cancel{V_0 t_b} - \frac{V_0 \pi}{2\omega} + \frac{\beta_i V_i^0 t_d}{2} - \frac{\beta_i V_i^0 t_b}{2} - \frac{\cancel{\beta_i V_i^0 \pi}}{4\omega} \end{aligned}$$

$$\frac{V_0 \pi}{\omega} = 2 \frac{\beta_i V_i^0}{2} [t_d - t_b]$$

\therefore we have

$$(\beta_i V_i^0) (t_d - t_b) = \frac{V_0 \pi}{\omega}$$

$$t_d - t_b = \frac{V_0 \pi}{\omega V_i^0 \beta_i}$$

$$\begin{aligned} t_d - t_b &= \frac{V_0}{V_0 V_i^0 \beta_i} \\ &= \frac{\pi V_0}{\omega V_i^0 \beta_i} \end{aligned}$$

Sub. eq (9) in eqⁿ (1)

$$\Delta V_L = V_0 \left[\frac{\pi V_0}{\omega V_1 \beta_1'} \right]$$

Output Power & Beam Loading.

The difference between the exit and entrance energies must be supplied by the buncher cavity to bunch the electron beam. Thus the electron beam is energised by energy of the cavity. This phenomenon is known as beamloading. The magnitude of induced current under beamloading is given by

$$i_{ind} = \beta_0 I_2$$

$$I_2 = 2 \beta_0 I_0 J_1(x)$$

The o/p power is given by the equation

$$P_{out} = \frac{(\beta_0 I_2)^2 R_{sh}}{2}$$

R_{sh} is the Resistance of the catcher cavity and R

$$R_{sh} = \frac{V_2}{\beta_0 I_2}$$

$$\therefore \text{output power} = \frac{(\beta_0 I_2)^2}{2} \times \frac{\beta_0 V_2}{\beta_0 I_2}$$

$$P_{out} = \frac{(\beta_0 I_2) V_2}{2}$$

The efficiency of the amplifier is defined by the equation

$$\eta = \frac{\beta_0 I_2 V_2}{2 V_0 I_0}$$

$$I_2 = 2 I_0 J_1(x)$$

$$\therefore \eta = \frac{\beta_0 V_2 J_1(x)}{V_0}$$

Mutual Conductance of Klystron

Mutual Conductance is the ratio of induced current to induced voltage. i.e.

$$\text{i.e. } G_M = \frac{I_{\text{induced}}}{V_1}$$

$$\therefore G_M = \frac{2\beta_0 I_0 J_1(x)}{V_1} \quad \text{--- (1)}$$

V_1 - Gap voltage or input cavity voltage for maximum o/p voltage

The voltage V_1 is expressed in terms of bunching parameter (x) as

$$\left(V_1 = \frac{2V_0 x}{\beta_0 \theta_0} \right) \quad \text{--- (2)}$$

where, θ_0 - Transit angle or dc transit angle.

$$\theta_0 = \frac{\omega L}{V_0}$$

Sub. eq(2) in eq(1)

$$G_M = \frac{I_0 J_1(x) \beta_0^2 \theta_0}{V_0 x}$$

Let, $\frac{I_0}{V_0} = G_0$ which is the beam Conductance.

$$\therefore G_M = \frac{\beta_0^2 G_0 J_1(x) \theta_0}{x}$$

$$\frac{G_M}{G_0} = \frac{\beta_0^2 \theta_0 J_1(x)}{x}$$

The term $\frac{G_M}{G_0}$ is known as normalized mutual Conductance.

$$\text{Voltage Gain, } A_V = \frac{V_2}{V_1}$$

$$V_2 = R_{sh} \beta_0 I_2$$

$$A_V = \beta_0 R_{sh} I_2 / V_1$$

Sub for V_1 from eq (2)

$$A_v = \frac{\beta_0 R_{sh} I_2}{2 V_0 X} \beta_0 \phi_0$$

$$= \frac{\beta_0^2 \phi_0 I_2 R_{sh}}{2 V_0 X}$$

$$I_2 = 2 I_0 J_1(x)$$

$$\therefore A_v = \frac{\beta_0^2 \phi_0 I_0 R_{sh} J_1(x)}{V_0 X}$$

$$= \frac{\beta_0^2 \phi_0 G_0 R_{sh} J_1(x)}{X}$$

$$\therefore A_v = G_m R_{sh}$$

Problem.

1. A two cavity klystron amplifier operates at a frequency of 5 GHz with DC beam voltage $V_0 = 10 \text{ kV}$. The cavity gap 'd' = 2 mm. The magnitude of gap voltage V_1 is 100 V. Find
 - i) uniform velocity of electrons before entering the gap (v_0)
 - ii) Transit time (τ)
 - iii) Transit angle (θ_0).

Sol Given that

$$V_0 = 10 \text{ kV}$$

$$V_1 = 100 \text{ V}$$

$$d = 2 \text{ mm}$$

$$f = 5 \text{ GHz}$$

i) uniform velocity of e^- entering the gap, $v_0 = 0.596 \times 10^8 \sqrt{V_0}$

$$\therefore V_0 = 0.596 \times 10^6 \sqrt{10 \times 10^3}$$

$$= 59.6 \times 10^6 \text{ V}$$

$$= 59.6 \text{ MW}$$

$$\text{ii) Transient time, } \tau = \frac{d}{V_0} = \frac{2 \times 10^{-3}}{59.6 \times 10^6} = \underline{\underline{3.355 \times 10^{-11} \text{ s}}}$$

$$\text{iii) } \theta_g = \omega \tau$$

$$= 2\pi f \tau$$

$$= 2\pi \times 5 \times 10^9 \times 3.355 \times 10^{-11}$$

$$= \underline{\underline{1.054 \text{ rad}}}$$

$$= 1.054 \times 57.29^\circ$$

$$= \underline{\underline{60.32^\circ}}$$

(1 microwave radian = 57.29)

2. An identical two cavity klystron amplifier operates at 4 GHz with $V_0 = 1 \text{ kV}$, $I_0 = 22 \text{ mA}$, cavity gap $d = 1 \text{ mm}$, drift space $L = 3 \text{ cm}$. If DC beam Conductance $G_0 = 0.25 \times 10^{-4} \text{ S}$ and Total Conductance $G_M = 0.3 \times 10^{-4} \text{ S}$. Calculate

i) Beam Velocity, V_0 .

ii) Transit angle, θ_g

iii) Beam Coupling Coefficient, β_0

iv) Input cavity voltage for max o/p voltage or gap voltage V_1 .

v) Voltage Gain.

vi) Efficiency.

Sol) Given

$$f = 4 \text{ GHz}, I_0, 22 \text{ mA}, L = 3 \text{ cm}; G_M = 0.3 \times 10^{-4} \text{ S}$$

$$V_0 = 1 \text{ kV}, d = 1 \text{ mm}, G_0, 0.25 \times 10^{-4} \text{ S}$$

$$V_0 = 0.596 \times 10^6 \sqrt{V_0}$$

$$= 18.84 \times 10^6 \text{ V}$$

Voltage Gain, $A_v = \frac{V_2}{V_1}$

$$A_v = \frac{\beta_o^2 \omega_o J_1(x) I_o R_{th}}{x V_o}$$

$$R_{th} = \frac{1}{\omega_o C_h}$$

$$Z = \frac{V_o}{I_o}$$

$$= \frac{1 \times 10^{-3}}{18.84 \times 10^6}$$

$$= 5.30 \times 10^{-11} \text{ s}$$

$$\theta_g = 2 \pi \times 9.25 \times 10^4 \times 5.30 \times 10^{-11}$$

$$= 1.332 \text{ rad}$$

$$= 76.31^\circ \quad 0.91j) \text{ yé } \%0.551 \times 0.551 \times$$

$$1.84 \times 1 \times 10^8 \times 0.55 \times 10^4$$

$$\text{Efficiency } \beta_o = \frac{\sin(\theta_g/2)}{\theta_g/2}$$

$$A_v \text{ in dB} = 10 \log 4.36$$

$$= \frac{0.929 \times 432.076 \times 0.582}{0.9 \times 10^3}$$

ii) Input capacitance, $V_1 = \frac{2 V_o x}{\beta_o \omega_o}$

$$\therefore V_1 = \frac{23.3\%}{\beta_o \omega_o} \times 2 V_o$$

$$\omega_o = \frac{\omega L}{V_o}$$

$$= \frac{2\pi \times 4 \times 10^9 \times 3 \times 10^{-2}}{18.84 \times 10^6}$$

$$= 40.020 \text{ rad}$$

$$V_1 = \frac{2 \times 1 \times 10^3 \times 1.84}{0.929 \times 40.02} = 98.98 \text{ V (approx.)}$$

X-branching parameter
with value

$$X = 1.84$$

\therefore the Bessel Functⁿ
 $J_1(x) = 0.582$

MODULE - 2

3/9/18

Monday

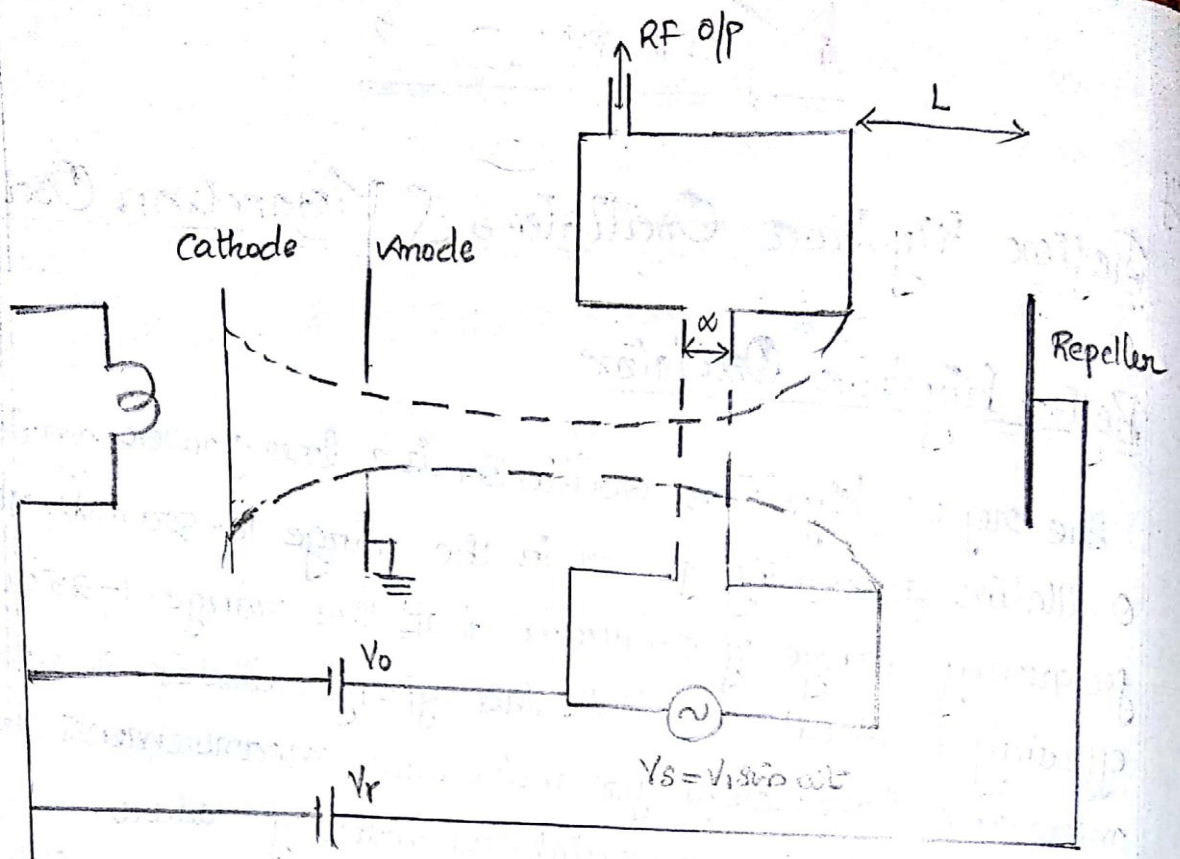
Reflex Klystron Oscillators & Magnetron Oscillators.

Reflex Klystron Oscillator

... klystron oscillator is a low power microwave

1. A two cavity klystron operates at 100 MHz with $I_0 = 3.6\text{ mA}$ and $V_0 = 10\text{ kV}$. The drift space length ' L ' = 2 cm and the total shunt capacitance $C_{sh} = 20\text{ pF}$. The beam coupling coefficient $\beta = 0.92$. Find the voltage gain.
2. A two cavity klystron amplifier has the following parameters
 $V_0 = 1000\text{ V}$, $R_0 = 40\text{ kV}$, $I_0 = 25\text{ mA}$, $F = 30\text{ MHz}$, $d = 1\text{ mm}$
 $L = 4\text{ cm}$, $R_{sh} = 30\text{ kV}$.
- Find the input gap voltage to give the maximum output voltage. (V)
 - Find the voltage gain (Av).
 - Find the efficiency of the amplifier.

the repeller space. Since the repeller space is at $-V_0$ potential, the electrons entering the repeller space are repelled back, on their return journey the electrons give up KE into the field. The electrons are finally collected by walls of the cavity by keeping unity gain and phaseshift of 2π . Sustained oscillations are obtained.



4/9/18

Tuesday Velocity Modulation

The uniform velocity with which the electrons enter the cavity is given by

$$V_0 = \sqrt{\frac{2eV_0}{m}}$$

where e - charge of electron
 m - mass of electron.

Sub. values of e and m .

$$V_0 = 0.596 \times 10^6 \sqrt{V_0} \text{ m/s. — (1)}$$

The velocity with which the electrons leave the cavity is given by

$$V(t) = V_0 \left[1 + \frac{\beta_1 V_1}{V_0} \sin(\omega t_1 - \theta_g/2) \right] \text{ — (2)}$$

The electric field inside the cavity is given by,

$$E = \frac{V_r + V_0 + V_1 \sin(\omega t_1)}{L}$$

where, V_r - repeller voltage

If $V_1 \sin(\omega t_1)$ is less than $V_r + V_0$, then

$$E = \frac{V_r + V_0}{L} \quad (3)$$

The force of electron is given

Substituting for E

$$F = m a = m \frac{d^2 z}{dt^2}$$

$$F = m a = m \frac{d^2 z}{dt^2} \quad (5)$$

Equating eq (4) & (5)

$$\frac{d^2 z}{dt^2} = -e \left[\frac{V_r + V_0}{L m} \right]$$

$$\frac{dz}{dt} = -e \left[\frac{V_r + V_0}{L m} \right] t + k_1$$

$$= -e \left[\frac{V_r + V_0}{L m} \right] [t - t_1] + k_1$$

The above eqn represents the velocity of the electron beam taking place inside the cavity. A is the ratio of hang

@e V60 mlye a Ob CEB Q

Output Power

For the electron beam to generate maximum amount of energy the beam should pass the cavity gap with less retardation. Thus, for max energy the transit angle is given by

$$\theta_0 = \omega(t_2 - t_1) = \omega T_0$$

$$\omega_0' = 2N\pi$$

where, N - the no: of modes

$$N = n - \frac{1}{4}$$

$$\therefore \omega_0' = 2\pi(n - \frac{1}{4})$$

$$= 2\pi n - \frac{\pi}{2} \quad \text{--- (1)}$$

The magnitude of current in the cavity is given by

$$I_2 = 2I_0 \beta_1 J_1(x') \quad \text{--- (2)}$$

The DC power supplied is given by

$$P_{dc} = V_0 I_0 \quad \text{--- (3)}$$

The AC power supplied is given by

$$P_{ac} = \frac{V_1 I_2}{2}$$

Sub for I_2 from eq (2)

$$P_{ac} = V_1 I_0 \beta_1 J_1(x') \quad \text{--- (4)}$$

Let, the bunching parameter of the reflex klystron is given by

$$* \quad x' = \frac{\beta_1 V_1 \omega_0'}{2V_0}$$

$$\therefore V_1 = \frac{2V_0 x'}{\beta_1 \omega_0'}$$

Sub for ω_0' from eq (1)

$$V_1 = \frac{2V_0 x'}{\beta_1 (2\pi n - \frac{\pi}{2})} \quad \text{--- (5)}$$

Sub eq (5) in (4)

$$P_{ac} = \frac{2V_0 x' I_0 J_1(x')}{2\pi n - \frac{\pi}{2}} \quad (6)$$

The efficiency of the klystron is given by:

$$\eta = \frac{P_{ac}}{P_{dc}}$$

Sub. values from eq (6) and (3)

$$\eta = \frac{2V_0 x' I_0 J_1(x')}{2\pi n - \frac{\pi}{2}} \times \frac{1}{V_0 I_0}$$

$$\boxed{\eta = \frac{2x' J_1(x')}{2\pi n - \frac{\pi}{2}}}$$

For a given beam voltage, V_0 the repeller voltage V_r is related to no. of modes as

$$* \frac{V_0}{(V_r + V_0)^2} = \frac{e}{m} \frac{(2\pi n - \pi/2)^2}{8\omega^2 L^2}$$

Admittance

The admittance of the klystron is the ratio of induced current to induced voltage.

$$\text{ie, } Y_e = \frac{i_2}{V_2}$$

$$\text{where } i_2 = 2I_0 \beta_i J_1(x') e^{-j\theta_0'}$$

$$* V_2 = I_2 R_{sh} \\ = \frac{2x' V_0 e^{-j\pi/2}}{\beta_i (2\pi n - \pi/2)}$$

Sub. i_2 and V_2 ,

$$Y_e = \frac{2I_0 \beta_i J_1(x') e^{-j\theta_0'}}{2x' V_0 e^{-j\pi/2} \times \beta_i (2\pi n - \pi/2)} \\ = \frac{I_0 \beta_i^2 J_1(x') e^{-j\theta_0'} (2\pi n - \pi/2)}{x' V_0 e^{-j\pi/2}}$$

$$\gamma_e = \frac{I_0 \beta_i^2 J_1(x') e^{j(\pi/2 - 0)} }{x' V_0 e^{j\pi/2}} = \frac{I_0 \beta_i^2 J_1(x')}{(2\pi n - \pi/2)}$$

Problem.

1. A reflex klystron operates under following conditions
 $V_0 = 600V$, $L = 1mm$, $R_{sh} = 15k\Omega$, $\frac{e}{m} = 1.769 \times 10^{11}$, $f = 90MHz$,
 $\eta = 2$.

i) Find the value of repeller voltage V_r

ii) Find I_0 for a voltage $V_r = 200V$

iii) Efficiency

So) Given

$$V_0 = 600V, \quad L = 1mm$$

$$R_{sh} = 15k\Omega, \quad \frac{e}{m} = 1.769 \times 10^{11}$$

$$f = 90MHz, \quad \eta = 2$$

we have

$$\frac{V_0}{(V_0 + V_r)^2} = \frac{e}{m} \frac{(2\pi n - \pi/2)^2}{8\omega^2 L^2}$$

$$V_r = I_2 R_{sh}$$

$$(V_0 + V_r)^2 = \frac{V_0 m}{e} \frac{8\omega^2 L^2}{(2\pi n - \pi/2)^2}$$

$$I_2 = 2 I_0 \beta_i J_1(x') R_{sh}$$

$$\beta_i = 1$$

$$(V_0 + V_r) = \sqrt{\frac{V_0 m}{e} \frac{8\omega^2 L^2}{(2\pi n - \pi/2)^2}}$$

$$\begin{aligned}
 V_x &= -V_0 + \sqrt{\frac{V_0 m}{e} \frac{8\omega^2 L^2}{(2\pi n - \frac{\pi}{2})^2}} \\
 &= -600 + \sqrt{\frac{600}{1.769 \times 10^{11}} \frac{8 \times (2 \times \pi \times 90 \times 10^9)^2 \times (1 \times 10^{-3})^2}{(2\pi(2) - \frac{\pi}{2})^2}} \\
 &= \underline{\underline{247.151V}}
 \end{aligned}$$

ii) Given
 $V_2 = 200V$

$$\therefore I_2 = \frac{V_2}{R_{sh}} = \frac{200}{15 \times 10^3} = \underline{\underline{0.0133A}}$$

also $I_2 = 2 I_0 \beta_i J_1(x')$ R_{sh}

$$\therefore I_0 = \frac{I_2}{2 \beta_i J_1(x') R_{sh}}$$

$$= \frac{0.0133}{2 \times 1 \times 0.582 \times 15 \times 10^3}$$

$$= \underline{\underline{7.617 \times 10^{-7}A}}$$

iii) Efficiency, $\eta = \frac{2x' J_1(x')}{2\pi n - \frac{\pi}{2}}$

$$= \frac{2 \times 1.84 \times 0.582}{2\pi \times 2 - \frac{\pi}{2}}$$

$$= 0.194$$

$$= \underline{\underline{19.4\%}}$$

6/1/18

Thursday

Magnetron Oscillators

A magnetron oscillator is used to generate high microwave power required in radar and communication systems. Magnetrons are crossfield tubes in which the magnetic field and electric field are perpendicular to each other. All magnetrons consist of some form of anode and cathode operated in DC magnetic field which is perpendicular to DC electric field. Due to this cross field, electrons emitted from cathode move in curved paths. If the magnetic field is strong enough the electrons will not arrive at the anode but return to cathode itself. Thus the anode current is cutoff. Magnetrons can be classified into 3.

i) Split anode magnetron

This type of magnetron uses negative resistance.

ii) Cyclotron freq magnetron

This magnetron operates under the influence of synchronisation between AC components of electric field and periodic oscillations of electrons in the direction parallel to the electric field.

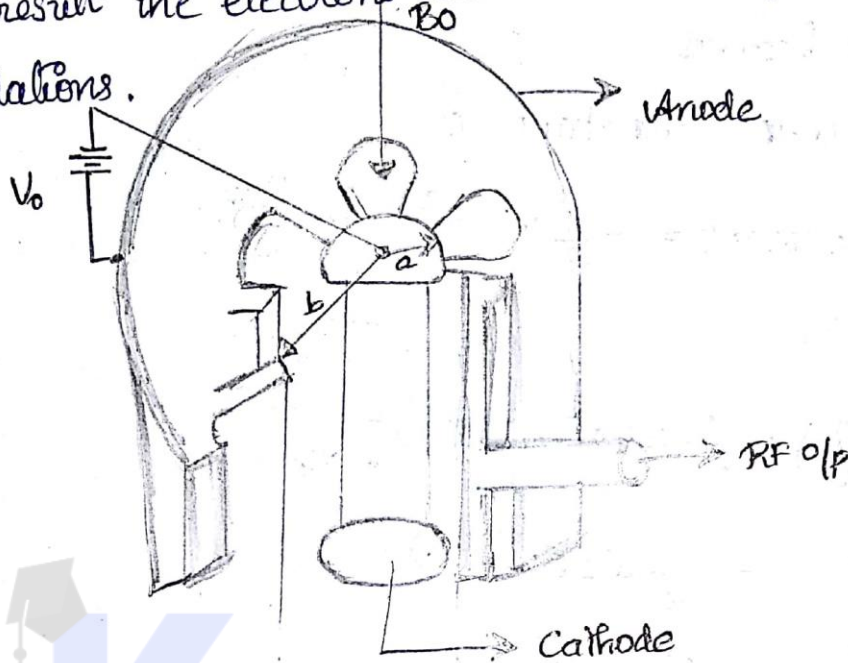
iii) Travelling wave magnetron

This type of magnetron depends on the interaction of electrons with travelling electromagnetic field.

Cylindrical Magnetron

It is also known as 'conventional magnetron'. In this type of magnetron several cavities are connected together. A DC voltage is applied between cathode and anode.

When the DC voltage and magnetic flux are adjusted properly the electrons will follow cycloidal paths in the space between anode and cathode. The electrons emitted from the cathode try to travel from to anode but due to the influence of cross field in the space between anode and cathode it experiences a repellent force, as a result the electrons take a curved path there by producing Oscillations.



B_0 - magnetic flux density.

a - radius of cathode silicon cylinders.

b - radius of anode cylinder.

V_0 - DC voltage

Equation of Electron Motion or Hull cut off Voltage Equation

A charged particle in motion in a magnetic field of flux density B experiences a force that is proportional to the charge, velocity, flux density and sine of angle between velocity and magnetic flux. The general equation for the motion of electron in terms of cylindrical coordinates is given as

$$r^2 \left(\frac{d\phi}{dt} \right) = \frac{1}{2} \omega_c r^2 + \text{Constant} \quad \text{--- (1)}$$

where ω_c is the ^{cyclotron} angular frequency given by

$$\omega_c = \frac{eBz}{m}$$

e - charge of electron

B_z - magnetic flux density

m - mass of electron.

let $r = a$ and $\frac{d\phi}{dt} = 0$

∴ eq (1) becomes

$$\frac{1}{2} \omega_c r^2 + \text{Constant} = 0$$

$$\text{Constant} = -\frac{1}{2} \omega_c a^2 \quad \text{--- (2)}$$

Sub eq (2) in (1)

$$r^2 \left(\frac{d\phi}{dt} \right) = \frac{1}{2} \omega_c r^2 - \frac{1}{2} \omega_c a^2$$

$$= \frac{1}{2} \omega_c (r^2 - a^2)$$

$$\frac{d\phi}{dt} = \frac{\omega_c}{2} \frac{(r^2 - a^2)}{r^2}$$

$$= \frac{\omega_c}{2} \left(1 - \frac{a^2}{r^2} \right) \quad \text{--- (3)}$$

The KE of electrons is given by *the learning companion*

$$\frac{1}{2} m v^2 = eV$$

$$v^2 = \frac{2eV}{m}$$

Considering r and ϕ components of electron velocity the above equation can be represented as

$$v_r^2 + v_\phi^2 = \frac{2eV}{m}$$

$$\left(\frac{dr}{dt} \right)^2 + \left(r \frac{d\phi}{dt} \right)^2 = \frac{2eV}{m} \quad \text{--- (4)}$$

Let $r = b$

$v = v_0$

$$\frac{dr}{dt} = 0$$

∴ the above eq (4) becomes .

$$\left(b \frac{d\phi}{dt}\right)^2 = \frac{2eV_0}{m} \quad (5)$$

Substituting $\frac{d\phi}{dt}$ from eq (3)

$$b^2 \left(\frac{\omega_c}{2} \left(1 - \frac{a^2}{b^2}\right) \right)^2 = \frac{2eV_0}{m}$$

Sub. $x=b$.

$$b^2 \left(\frac{\omega_c}{2} \left(1 - \frac{a^2}{b^2}\right) \right)^2 = \frac{2eV_0}{m}$$

$$\frac{b^2 \omega_c^2}{4} \left[1 - \frac{a^2}{b^2}\right]^2 = \frac{2eV_0}{m}$$

$$b^2 \omega_c^2 \left(1 - \frac{a^2}{b^2}\right)^2 = \frac{8eV_0}{m}$$

Sub. $\omega_c = \frac{eB_z}{m}$

$$b^2 \left(\frac{eB_z}{m} \right)^2 \left(1 - \frac{a^2}{b^2}\right)^2 = \frac{8eV_0}{m}$$

$$\frac{b^2 e^2}{m^2} \left(1 - \frac{a^2}{b^2}\right)^2 B_z^2 = \frac{8eV_0}{m}$$

$$B_z^2 = \frac{8eV_0}{m} \times \frac{m^2}{b^2 e^2} \frac{1}{\left(1 - \frac{a^2}{b^2}\right)^2}$$

$$B_z = \sqrt{\frac{8eV_0 m}{b^2 e^2} \frac{1}{\left(1 - \frac{a^2}{b^2}\right)^2}}$$

This magnetic field intensity is known as cut off flux density, B_{oc}

$$B_{oc} = \sqrt{\frac{8V_0 m}{eb^2}} \sqrt{\frac{1}{1 - \frac{a^2}{b^2}}}$$

$$B_{oc} = \sqrt{\frac{8V_0 m}{eb^2} \frac{1}{\left(1 - \frac{a^2}{b^2}\right)^2}}$$

$$V_0 = \frac{m}{8e} \frac{b^2 e^2}{m^2} \left(1 - \frac{a^2}{b^2}\right)^2 B_z^2$$

$$V_{oc} = \frac{b^2 e}{8m} B_z^2 \left(1 - \frac{a^2}{b^2}\right)^2$$

This equation is known as 'Hull cut off voltage equation'.

Cyclotron Angular Frequency

The cyclotron angular frequency of cylindrical magnetron is given by, $\omega_c = \frac{eB}{m}$

The period of oscillation is given by

$$T = \frac{2\pi}{\omega_c}$$

$$\therefore T = \frac{2\pi m}{eB}$$

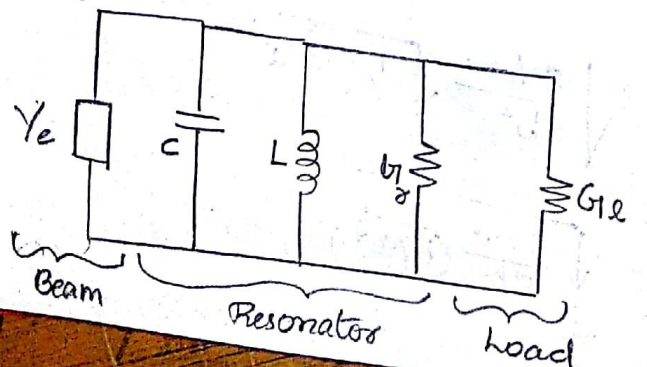
The phase shift is given by

$$\phi = \frac{2\pi m}{N}$$

where, N is the no. of cycles.

Output Power & Efficiency.

The efficiency and op. power of the magnetron depend on the structure and power supply. The equivalent circuit of magnetron is



Y_e - admittance

C - capacitance

L - Inductance

G_r - Resonator Conductance

G_L - load Conductance

→ The unloaded quality factor is given by,

$$Q_{un} = \frac{\omega_0 C}{G_r}$$

The loaded quality factor is given by,

$$Q_L = \frac{\omega_0 C}{G_r + G_L}$$

The external quality factor,

$$Q_{ext} = \frac{\omega_0 C}{G_L}$$

→ The circuit efficiency, $\eta_c = \frac{1}{1 + \frac{Q_{ext}}{Q_{un}}}$

→ Electronic efficiency, $\eta_e = \frac{V_o I_o - P_{lost}}{V_o I_o}$

where V_o - anode voltage
 I_o - anode current or beam current.

P_{lost} - power lost in the anode.

Problem.

1. A cylindrical magnetron is operated in following parameters
anode voltage $V_o = 26 \text{ kV}$, beam current $I_o = 27 \text{ A}$, magnetic flux density $B_z = 0.336 \text{ weber/m}^2$, $a = 5 \text{ cm}$, $b = 10 \text{ cm}$. find
 - i) Cyclotron angular frequency
 - ii) Hull cut off voltage
 - iii) Cut off magnetic flux density B_{oc} .

Sol. Given

$$V_0 = 26 \text{ kV}$$

$$I_0 = 27 \text{ A}$$

$$B_z = 0.336 \text{ wb/m}^2$$

$$a = 5 \text{ cm}$$

$$b = 10 \text{ cm}$$

$$\begin{aligned} \text{i) } \omega_c &= \frac{eB}{m} \\ &= \frac{1.6 \times 10^{-19} \times 0.336}{9.1 \times 10^{-31}} \\ &= \underline{\underline{5.907 \times 10^{10} \text{ Hz rad}}} \end{aligned}$$

$$\begin{aligned} \text{ii) } V_0 &= \frac{b^2 e}{8m} B_z^2 \left(1 - \frac{a^2}{b^2}\right)^2 \\ &= \frac{(10 \times 10^{-2})^2 \times 1.6 \times 10^{-19} (0.336)^2}{8 \times 9.1 \times 10^{-31}} \left(1 - \frac{(5 \times 10^{-2})^2}{(10 \times 10^{-2})^2}\right)^2 \\ &= \underline{\underline{12.862 \times 10^6}} \quad \underline{\underline{12.95 \text{ MV}}} \\ &= \underline{\underline{12.862 \text{ MV}}} \end{aligned}$$

$$\begin{aligned} \text{iii) } B_{oc} &= \sqrt{\frac{8V_0 m}{e b^2} \left(1 - \frac{a^2}{b^2}\right)^2} \\ &= \underline{\underline{0.3359}} \quad \underline{\underline{0.0145}} = \underline{\underline{14.5 \times 10^3}} \end{aligned}$$

Ans

The magnetron has following operating parameters
anode voltage $V_0 = 5.5 \text{ kV}$, beam current $I_0 = 4.5 \text{ A}$

MODULE - 3

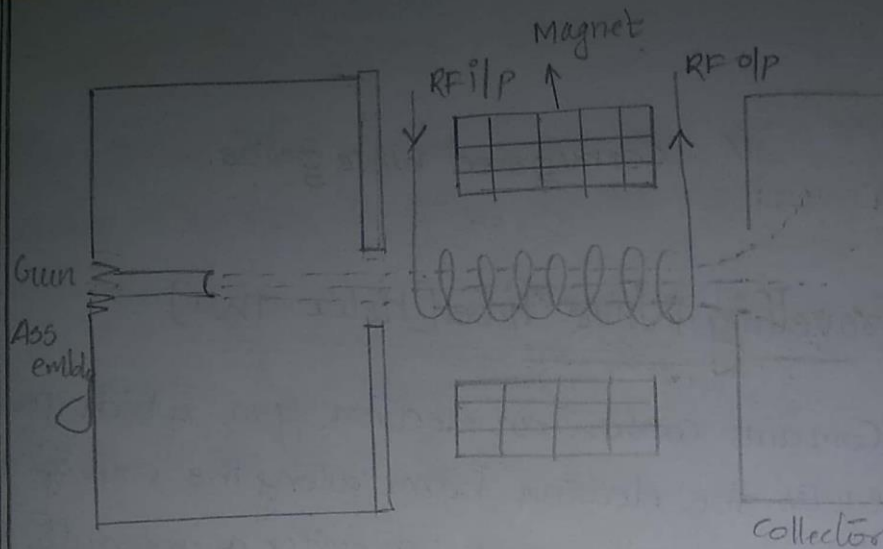
Travelling Wave Tubes & Microwave Measurements

Travelling Wave Tube (TWT)

TWT is a high gain, low noise, wide band microwave amplifier. They work on the principle of interaction between electron beam & RF field. In TWT slow wave structures are used instead of resonant cavities as in klystrons.

Basic Operation

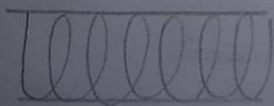
TWT has an electron gun which produces and accelerates the electron beam along the axis of the tube. Surrounding magnet provides a magnetic field along the axis of the tube to focus the electron beam. The slow wave structure at the center of the tube provides a low impedance transmission path for the RF energy within the tube. The applied i/p signal which is to be amplified propagates along the slow wave structure. It produces an electric field at the center of the helix directed along the axis. When the electron beam enters the tube the axial electric field and electron beam are interacted. The electrons entering the slow wave structure at zero electric field are not affected. Those entering at accelerating field are accelerated and those entering at retarding field are decelerated. Thus, velocity modulation takes place & electrons are bunched together and are collected at the collector. The microwave energy of electrons is transferred to slow wave structure & thus the signal is amplified.



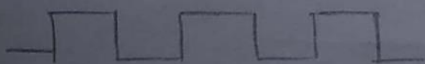
Slow Wave Structures.

They are special circuits that are used in microwave tubes to reduce the wave velocities in certain directions so that the electron beam & signal can interact. They reduce the velocity of the wave so that electron beam efficiently interact with it. They are non resonant periodic circuits. In a normal resonant circuit as the authorative operating freq is increased both the inductance & capacitance of the resonant circuit must be decreased to maintain resonance at the operating frequency. This limits the gain & reduces the op. Slow wave structures compensate this limitation and produces large op over a wide bandwidth.

→ Various types of slow wave structures are:



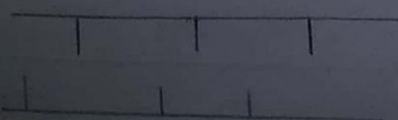
Helical Line



Folded Back - Line.

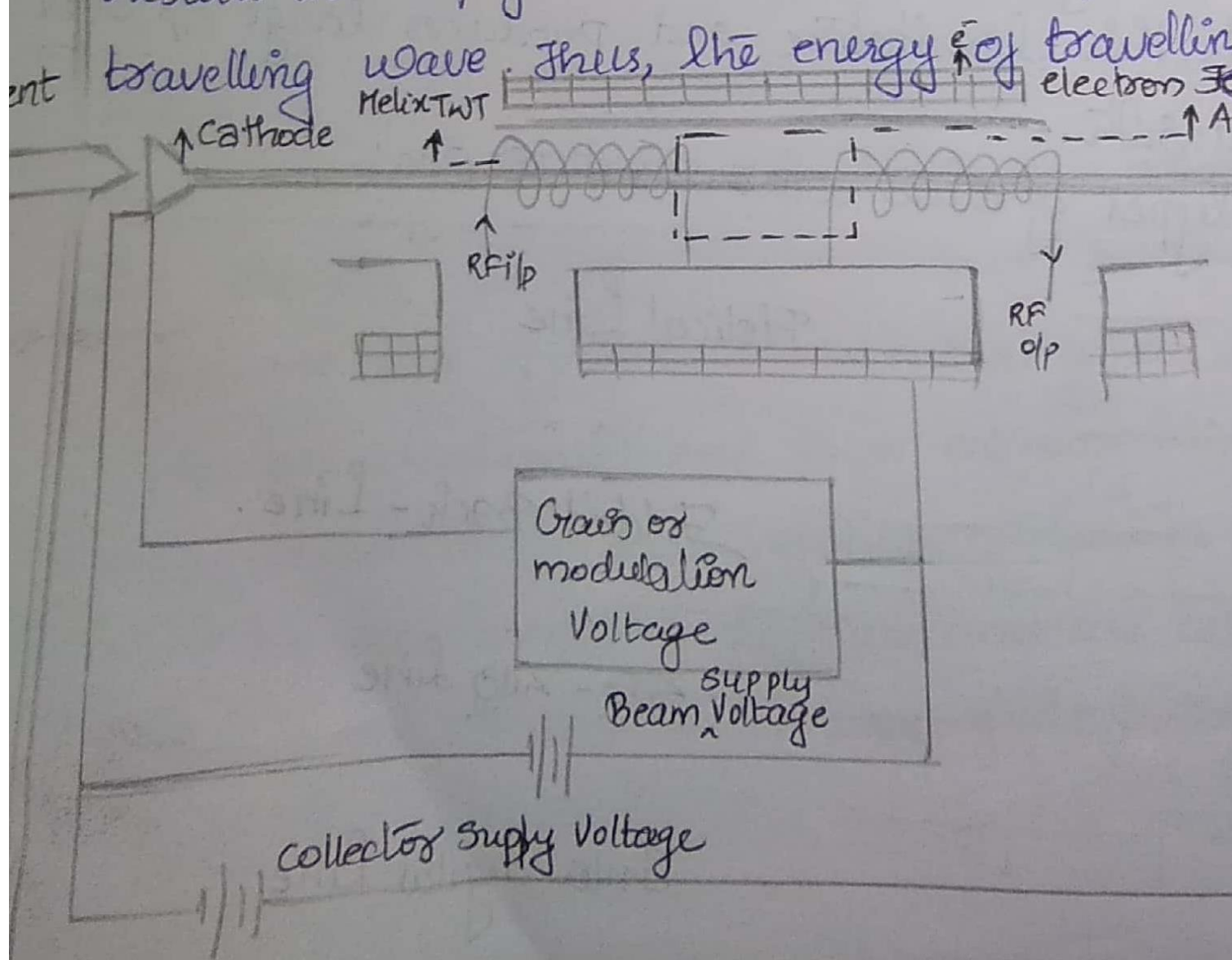


Zig-zag Line



Inter digital Line

straight beam. A longitudinal helix is placed at the center of the tube which provides a transmission path. The RF i/p & o/p are coupled from the helix by directional couplers. The tube is designed with helix delay structure to make the wave below or equal to the speed of electron of the beam are accelerated by interacting of the electric field through velocity modulation results in amplification & the electrons give up



Amplification process

The phase shift of the fundamental wave on the helical slow wave structure is given by the eqn

$$\phi_0 = \beta_z L$$

where, β_z - phase constant,

→ which is defined as $\beta_z = \frac{\omega}{v_p}$

L is the period of the signal.

→ The eqn for motion of electron is given by

$$m \frac{dv}{dt} = -e E_1 \sin[\omega t - \beta_z z] \quad \text{--- (1)}$$

Let the velocity of electron be defined as

$$v = v_0 + v_e \cos(\omega_e t + \phi_e)$$

$$\therefore \frac{dv}{dt} = \frac{dv_e}{dt} - v_e \sin(\omega_e t + \phi_e) \omega_e \quad \text{--- (2)}$$

Sub. eq (2) in (1)

$$\therefore -m v_e \sin(\omega_e t + \phi_e) \omega_e = -e E_1 \sin[\omega t - \beta_z z] \quad \text{--- (3)}$$

v_e is the magnitude of velocity fluctuation, ω_e is the angular velocity, ϕ_e - phase angle fluctuation.

Let, the distance ' z ' travelled by the electron is defined as

$$z = v_0(t - t_0) \quad \text{--- (4)}$$

where, v_0 is the 'dc electron velocity'

Sub eq (4) in (3)

$$\begin{aligned} m v_e \omega_e \sin(\omega_e t + \phi_e) &= e E_1 \sin[\omega t - \beta_z v_0(t - t_0)] \\ &= e E_1 \sin[\omega t - \beta_z v_0 t + \beta_z v_0 t_0] \end{aligned}$$

Equating LHS & RHS

$$m v_e u_e = e E_1$$

$$\therefore v_e = \frac{e E_1}{m u_e}$$

ie, the magnitude of velocity fluctuation is directly proportional to magnitude of electric field, E_1 .

equating terms of +

$$u_e = \omega - \beta_z v_0$$

$$\omega = \beta_z v_p$$

$$u_e = \beta_z v_p - \beta_z v_0$$

$$u_e = \beta_z (v_p - v_0)$$

where v_p - axial velocity

ie, the angular velocity u_e is proportional to difference betⁿ axial & dc velocities.

Convection Current

When the e^- beam passes through the electric field, it induces a current. This current is known as convection current.

The electron velocity, charge density, current density and axial electric field are given by the equations.

$$v_0 = v_0 + v_1 e^{j\omega t - \gamma z} \quad \text{--- (a)}$$

$$\rho = \rho_0 + \rho_1 e^{j\omega t - \gamma z} \quad \text{--- (b)}$$

$$J = -J_0 + J_1 e^{j\omega t - \gamma z} \quad \text{--- (c)}$$

$$E_z = E_1 e^{j\omega t - \gamma z} \quad \text{--- (d)}$$

For a small signal the current density of electrons given by $J = \rho v$.

Sub for ρ & v from eqⁿs (a) & (b)

$$J = (P_0 + P_1 e^{j\omega t - r_1 z}) (C V_0 + V_1 e^{j\omega t - r_1 z})$$

Since the signal is very small the term $e^{j\omega t - r_1 z}$ is neglected.

$$\therefore J = C P_0 + P_1 (C V_0 + V_1)$$

$$J = P_0 V_0 + P_0 V_1 + P_1 V_0 + P_1 V_1$$

$$\text{Let } P_1 V_1 = 0$$

$$\therefore J_1 = P_0 V_0 + P_0 V_1 + P_1 V_0$$

Sub. for J from eqn (c) the above eqn becomes

$$-J_0 + J_1 e^{j\omega t - r_1 z} = P_0 V_0 + P_0 V_1 + P_1 V_0$$

$$\text{Let } P_0 V_0 = -J_0 \text{ \& } J_1 = P_0 V_1 + P_1 V_0 \text{ — (e)}$$

The axial electric field effects the velocity of electrons as

$$\frac{dv}{dt} = \frac{-e}{m} E_1 e^{j\omega t - r_1 z} \text{ — (1)}$$

Let the change in velocity be defined as

$$\frac{dv}{dt} = \frac{\partial v}{\partial t} + \frac{\partial z}{\partial t} \frac{\partial v}{\partial z}$$

Sub. for v from eqn (a)

$$\frac{dv}{dt} = \frac{\partial}{\partial t} (C V_0 + V_1 e^{j\omega t - r_1 z}) + \frac{\partial z}{\partial t} \frac{\partial}{\partial z} (C V_0 + V_1 e^{j\omega t - r_1 z})$$

$$\begin{aligned} \frac{dv}{dt} &= j\omega V_1 e^{j\omega t - r_1 z} - V_1 r_1 e^{j\omega t - r_1 z} \cdot \frac{\partial z}{\partial t} \\ &= V_1 e^{j\omega t - r_1 z} (j\omega - r_1 \frac{\partial z}{\partial t}) \end{aligned}$$

$$\text{Let } \frac{\partial z}{\partial t} = V_0$$

$$\therefore \frac{dv}{dt} = V_1 e^{j\omega t - r_1 z} (j\omega - r_1 V_0) \text{ — (2)}$$

Sub. eqn (2) in (1)

$$V_1 e^{j\omega t - r_1 z} (j\omega - r_1 V_0) = \frac{-e}{m} E_1 e^{j\omega t - r_1 z}$$

$$V_1 e^{j\omega t - \gamma z} (j\omega - \gamma v_0) = \frac{-e}{m} E_1$$

$$V_1 = \frac{-e}{m(j\omega - \gamma v_0)} E_1 \quad \text{--- (3)}$$

eqn (3) defines the change in electron velocity w.r. to electric field E_1 .

By law of conservation of electric charge the continuity eqn can be written as.

$$\nabla \cdot J = -\frac{\partial \rho}{\partial t} \quad \text{--- (4)}$$

$$\nabla = \frac{\partial}{\partial z} + \frac{\partial}{\partial y} + \frac{\partial}{\partial x}$$

Consider only the z component

$$\nabla = \frac{\partial}{\partial z}$$

$$\therefore \nabla \cdot J = \frac{\partial (J)}{\partial z}$$

Sub. for J from eq (3), we get

$$\nabla \cdot J = \frac{\partial}{\partial z} (-J_0 + J_1 e^{j\omega t - \gamma z})$$

$$\nabla \cdot J = -\gamma J_1 e^{j\omega t - \gamma z} \quad \text{--- (5)}$$

from eq (b)

$$-\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial t} (C_0 + \rho_1 e^{j\omega t - \gamma z})$$

$$-\frac{\partial \rho}{\partial t} = -\rho_1 j\omega e^{j\omega t - \gamma z} \quad \text{--- (6)}$$

eq sub. eq (5) & (6) in eqn (4)

$$-\gamma J_1 e^{j\omega t - \gamma z} = -\rho_1 j\omega e^{j\omega t - \gamma z}$$

$$\therefore -\gamma J_1 = -\rho_1 j\omega$$

$$\frac{r_{J_1}}{j\omega} = p_1$$

$$\text{i.e. } \left(p_1 = \frac{r_{J_1}}{j\omega} \right) \quad (7)$$

eqn (7) defines the change in charge density ρ with resp. to current density J .

Consider, eqn (6)

$$J_1 = \rho_0 v_1 + p_1 v_0$$

Sub. for v_1 from eqn (3) & p_1 from eqn (7)

$$\therefore J_1 = \rho_0 \left(\frac{-e}{m j \omega - r v_0} E_1 \right) + \left(\frac{r J_1}{j \omega} \right) v_0$$

$$J_1 - \frac{r v_0}{j \omega} J_1 = \frac{-\rho_0 e E_1}{m j \omega - r v_0}$$

$$\frac{1}{j} = -j$$

$$J_1 \left(1 + \frac{r v_0 j}{\omega} \right) = \frac{-\rho_0 e E_1}{m j \omega - r v_0}$$

$$J_1 \left[\frac{\omega + r v_0 j}{\omega} \right] = \frac{-\rho_0 e E_1}{m j \omega - r v_0}$$

$$\therefore J_1 = \frac{-\rho_0 e E_1 \omega}{m j \omega - r v_0 (\omega + r v_0 j)}$$

Multiplying numerator & denominator by v_0

$$J_1 = \frac{-\rho_0 e E_1 \omega v_0}{m j \omega - r v_0 (\omega + r v_0 j) v_0}$$

we have,

$$J_0 = -\rho_0 v_0$$

$$\beta_e = \omega_0 / v_0$$

$$\therefore J_1 = \frac{J_0 e E_1 \beta_e}{m j \omega - r v_0 (\omega + r v_0 j)}$$

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multiplying numerator & denominator with j

$$T_1 = \frac{J_0 e E_1 \beta e j}{jm(j\omega - rV_0)(\omega + rV_0j)}$$

$$\therefore T_1 = \frac{J_0 e E_1 \beta e j}{jm(-\omega - rV_0j)(\omega + rV_0j)}$$

$$= \frac{J_0 e E_1 \beta e j}{m(-\omega^2 - \omega rV_0j - rV_0j\omega + r^2V_0^2)}$$

$$= \frac{J_0 e E_1 \beta e j}{m(-\omega^2 - 2\omega rV_0j + (rV_0)^2)}$$

$$= \frac{J_0 e E_1 \beta e j}{m(j\omega)^2 - 2\omega rV_0j + (rV_0)^2}$$

$$= \frac{J_0 e E_1 \beta e j}{m(j\omega + rV_0)^2}$$

$$= \frac{J_0 e E_1 \beta e j}{V_0^2 m \left(j \frac{\omega}{V_0} - r \right)^2}$$

$$= \frac{J_0 e E_1 \beta e j}{V_0^2 m (j\beta e - r)^2} \quad \because \frac{\omega}{V_0} = \beta e$$

The uniform velocity V_0 is defined by the eqn

$$V_0 = \sqrt{\frac{2eV_0}{m}}$$

$$V_0^2 = \frac{2eV_0}{m}$$

$$\frac{e}{mV_0^2} = \frac{1}{2V_0}$$

$$\therefore J_1 = \frac{J_0 E_1 \beta e^j}{2V_0 (j\beta e - r)^2}$$

when the electric field is uniform $J_1 = I i$

$$\text{hence, } i = \frac{J_0 E_1 \beta e^j}{2V_0 (j\beta e - r)^2}$$

This is the equation for convection current. This eqⁿ is also known as 'electronic equation' as it determines the current induced by the electric field

Axial Electric Field.

It defines how the electric field changes with convection current. Let

Let 'L' be the inductance, C be the capacitance, ω be the alternating current, V be the alternating voltage & i be the convection current. The current flowing is defined by the eqⁿ.

$$\frac{\partial I}{\partial z} = -C \frac{\partial V}{\partial t} - \frac{\partial i}{\partial z} \quad \text{--- (1)}$$

$$\text{Let, } \frac{\partial}{\partial z} = -\gamma \quad \& \quad \frac{\partial}{\partial t} = j\omega$$

\therefore eqⁿ (1) becomes

$$-\gamma I = -j\omega CV + \gamma i \quad \text{--- (2)}$$

The voltage across the electric field is defined as

$$\frac{\partial V}{\partial z} = -L \frac{\partial I}{\partial t}$$

$$-\gamma V = -j\omega LI$$

$$I = \frac{\gamma V}{j\omega L} \quad \text{--- (3)}$$

Sub eq (3) in (2)

$$\frac{-\gamma^2 V}{j\omega L} = -j\omega CV + \gamma i$$

$$V^2 = -j\omega L [-j\omega C V + I] -$$

$$= -CV\omega^2 - j\omega L I$$

when the convection current $I = 0$,

$$V^2 = -CV\omega^2$$

$$V^2 = -C\omega^2 L$$

$$V = \sqrt{-CL\omega^2}$$

$$= j\omega\sqrt{LC}$$

This eqⁿ represents propagation const. V for an ideal transmission line

$$\text{i.e. } V_0 = j\omega\sqrt{LC}$$

$$V_0^2 = -\omega^2 LC \quad (4)$$

when the convection current is not zero.

$$V^2 = -\omega^2 CV - Ij\omega L$$

$$V^2 + \omega^2 CLV = -Ij\omega L$$

$$V(V^2 + \omega^2 CL) = -Ij\omega L$$

$$V = \frac{-Ij\omega L}{(V^2 + \omega^2 CL)}$$

Sub. eqⁿ (4)

$$V = \frac{-I\omega L j}{-\omega^2 (V^2 - V_0^2)} \quad (5)$$

Characteristic impedance

$$Z_0 = \sqrt{\frac{L}{C}}$$

multiplying both sides by V_0

$$V_0 Z_0 = V_0 \sqrt{\frac{L}{C}}$$

Sub. for V_0 from eq (4)

$$\begin{aligned} V_0 Z_0 &= j\omega \sqrt{LC} \sqrt{\frac{L}{C}} \\ &= j\omega L \end{aligned}$$

Sub. the value of $Z_0 V_0$ in eqⁿ (5).

$$V = \frac{-jI' V_0 Z_0}{(V'^2 - V_0^2)}$$

The electric field is defined as

$$E_z = -\nabla \cdot \Phi B$$

$$E_z = -\nabla \cdot V$$

$$E_z = -\frac{\partial V}{\partial z}$$

$$\text{we have } +\frac{\partial}{\partial z} = -V'$$

$$E_z = V'V$$

Sub. for V .

$$\therefore E_z = \frac{-V'^2 Z_0 V_0 I}{(V'^2 - V_0^2)}$$

This eqⁿ is called circuit equation as it determines the electric field from the convection current I .

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Wave Modes.

→ The solution for propagation constant V' in the electronic & circuit equations provides four distinct solutions. These 4 solutions represent four modes of propagation in a travelling wave tube. The values of four propagation constants are given by

$$V_1 = -\beta_e c \frac{\sqrt{3}}{2} + j\beta_e \left[1 + \frac{c}{2}\right] \quad T \rightarrow e^- \quad v \downarrow$$

$$V_2 = \beta_e c \frac{\sqrt{3}}{2} + j\beta_e \left[1 + \frac{c}{2}\right] \quad e \rightarrow T \quad v \downarrow$$

$$V_3 = j\beta_e [1 - c] \quad v \uparrow$$

$$V_4 = -j\beta_e \left[1 - \frac{c^3}{4}\right] \quad \text{backward wave.}$$

The wave corresponding to V_1 is a forward wave and its amplitude grows exponentially with distance.

The wave corresponding to V_2 is a forward wave whose amplitude decays exponentially. The wave corresponding to V_3 is a forward wave whose amplitude remains constant. The wave corresponding to V_4 is a backward wave whose amplitude remains constant. The growing wave propagates at a velocity slightly lower than electron velocity & energy flows from electron beam to travelling wave. The decaying wave propagates in the same manner as that of the growing wave but the energy flows from travelling wave to electron beam. The constant amplitude wave travels ^{with a} velocity higher than electron velocity and no energy transfer occurs. The backward wave propagates in -ve z direction with a velocity higher than electron velocity.

Gain characteristics.

The gain of the travelling wave tube is defined as

$$A_p = 10 \log \left| \frac{O/P \text{ Vtg}}{I/P \text{ Vtg}} \right|^2$$

$$A_p = -9.54 + 47.3 N C \text{ dB.}$$

where, N - length

C - Grain parameter.

Comparison betⁿ klystron & Travelling wave Tube.

<u>klystron</u>	<u>Travelling wave tube</u>
i) linear beam device	i) linear beam device.
ii) uses Cavities for Operation	ii) uses slow wave structures for operation.
iii) Narrow band device	iii) wide band device.

Microwave Measurements.

1) Power Measurements

- Power is defined as quantity of energy dissipated..
- The microwave power meter has a power sensor which converts the microwave power into heat energy.
- The rise in temperature provides a change in electrical parameters, resulting in o/p current.
- The 3 commonly used power measurement methods are

i) Schottky Barrier Diode Sensor.

It is used as a square law detector whose o/p is proportional to I_p

ii) Bolometer Sensor

It is a power sensor whose resistance changes with temp. The two common types of bolometer sensors are barretter & Thermistor. The barretter has positive temp. coefficient of resistance. Thermistor has -ve temp. coefficient of resistance.

iii) Thermocouple Sensor

A thermocouple is a junction of two dissimilar metals

or semiconductors. The EMF generated by the thermocouple when it is heated is proportional to microwave power to be measured.

ii) Impedance Measurement

a) Slotted Line Method

In this method the impedance of the load can be measured by measuring the phase angle & reflection coefficient. The load impedance is given by

$$Z_L = Z_0 \frac{1 + \Gamma_L}{1 - \Gamma_L}$$

where, $\Gamma_L = \rho_L e^{j\phi_L}$

b) Reflectometer Method

The unknown impedance can be measured by the equation

$$Z_L = \frac{A \Gamma_L + B}{C \Gamma_L + D}$$

where, A, B, C & D are the signal amplitudes.

iii) Frequency Measurement

a) Down Conversion Method

This method uses a heterodyne converter. It converts unknown frequency by mixing with a known frequency such that the difference between frequencies is amplified and measured.

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This method uses a heterodyne converter. It converts unknown frequency by mixing with a known frequency such that the difference between frequencies is amplified and measured.

24/9/18

Thursday

MODULE 4

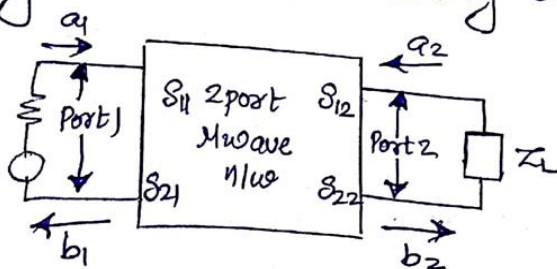
Microwave Hybrid Circuits & Directional Couplers

A microwave nlw or microwave hybrid circuit consist of several microwave devices such as sources, attenuators, filters, amplifiers etc coupled together by transmission lines for the transmission of microwave signal. The point of interconnection of two or more devices is known as a junction. The measurement of Z, Y, h & ABCD parameter is difficult at microwave frequencies due to following reasons.

- 1) Non-availability of voltage & current measuring equipment
- 2) Short circuit not easily achieved for wide range of frequency
- 3) Presence of active devices make the circuit unstable so microwave circuits are analyzed using scattering parameters or 'S' matrix. S matrix relates the amplitude of reflected waves with incident waves

Scattering matrix or 'S' matrix

It is a square matrix which gives all the combinations of power relationship between i/p & o/p ports of a microwave junction. The elements of 'S' matrix are known as scattering parameters or scattering coefficients.



Consider the microwave 2 port n/w. a_1 is the amplitude of incident wave at port 1.

a_2 - amplitude of incident wave at port 2.

b_1 - amplitude of reflected wave at port 1.

b_2 - amplitude of reflected wave at port 2.

The incident and ~~reflected~~ reflected waves can be related using 'S' matrix as

$$[b] = [S][a]$$

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$b_1 = S_{11} a_1 + S_{12} a_2$$

$$b_2 = S_{21} a_1 + S_{22} a_2$$

S_{11} - reflection coefficient at port 1 when $a_2 = 0$.

$$\Rightarrow S_{11} = \frac{b_1}{a_1} \bigg|_{a_2=0}$$

S_{22} is the reflection coefficient at port 2 when $a_1 = 0$

$$\Rightarrow S_{22} = \frac{b_2}{a_2} \bigg|_{a_1=0}$$

S_{12} is the attenuation of wave travelling from port 2 to port 1 when $a_1 = 0$.

$$\Rightarrow S_{12} = \frac{b_1}{a_2} \bigg|_{a_1=0}$$

S_{21} is the attenuation of wave travelling from port 1 to port 2 with $a_2 = 0$.

$$\Rightarrow S_{21} = \frac{b_2}{a_1} \bigg|_{a_2=0}$$

In a microwave n/w if the incident power is P_i , reflected power is P_r , or power is P_o . then, the losses defined are

$$i) \text{ Insertion loss} = 10 \log \left(\frac{P_i}{P_o} \right)$$

$$ii) \text{ Transmission loss or attenuation} = 10 \log \left(\frac{P_i - P_r}{P_o} \right)$$

$$iii) \text{ Reflection loss} = 10 \log \left(\frac{P_o}{P_i - P_r} \right)$$

$$iv) \text{ Return loss} = 10 \log \left(\frac{P_i}{P_r} \right)$$

Properties of S matrix

- 'S' matrix is always a square matrix of order $n \times n$.
- Under perfect match condition the diagonal elements of 'S' matrix are zero.
- 'S' matrix is always symmetric. i.e., $S_{ij} = S_{ji}$.
- 'S' matrix is an unitary matrix. i.e., $[S][S]^* = I$
I is an identity matrix.
- The sum of product of each term of any row or column multiplied by complex conjugate of corresponding term of another row or column is zero.

$$\sum_{j=1}^n S_{ij} S_{ik}^* = 0$$

- In a two port n/w if the reference plane are shifted from one and two to 1' and 2' the new S matrix is given by

$$[S'] = \begin{bmatrix} e^{-j\phi_1} & 0 \\ 0 & e^{-j\phi_2} \end{bmatrix} [S] \begin{bmatrix} e^{j\phi_1} & 0 \\ 0 & e^{j\phi_2} \end{bmatrix}$$

This property is known as phase shift property.

- Since S matrix is symmetric $[S]^T = [S]$

→ Since the E plane Tee is a 3 port network. The general S matrix is represented as

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \quad \text{--- (1)}$$

The wave fed into port 3 appears at port 1 and port 2 with equal magnitude & opposite phase.

$$\text{i.e., } S_{13} = -S_{23} \quad \text{--- (2)}$$

$$\text{If port 3 is matched, } S_{33} = 0. \quad \text{--- (3)}$$

by the property of Symmetry

$$\left. \begin{aligned} S_{12} &= S_{21} \\ S_{13} &= S_{31} \\ S_{23} &= S_{32} \end{aligned} \right\} \quad \text{--- (4)}$$

applying, eqn (2), (3) & (4) in eqn (1)

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & -S_{13} \\ S_{13} & -S_{13} & 0 \end{bmatrix} \quad \text{--- (5)}$$

By, unitary property matrix $[S][S]^* = I$

i.e

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & -S_{13} \\ S_{13} & -S_{13} & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{12}^* & S_{22}^* & -S_{13}^* \\ S_{13}^* & -S_{13}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{--- (6)}$$

$$\text{R1C1} \Rightarrow S_{11} S_{11}^* + S_{12} S_{12}^* + S_{13} S_{13}^* = 1$$

$$\Rightarrow |S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1 \quad \text{--- (6)}$$

$$R_2 C_2 \Rightarrow S_{12} S_{12}^* + S_{22} S_{22}^* + S_{13} S_{13}^* = 1$$

$$\Rightarrow |S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 = 1 \quad \text{--- (7)}$$

$$R_3 C_3 \Rightarrow |S_{13}|^2 + |S_{13}|^2 + 0 = 1$$

$$\Rightarrow 2|S_{13}|^2 = 1$$

$$\Rightarrow |S_{13}|^2 = \frac{1}{2}$$

$$S_{13} = \frac{1}{\sqrt{2}} \quad \text{--- (8)}$$

$$R_3 C_4 \Rightarrow S_{13} S_{11}^* + -S_{13} S_{12}^* = 0$$

$$\Rightarrow S_{13} [S_{11}^* - S_{12}^*] = 0$$

$$\Rightarrow S_{13} = 0$$

$$\text{ie, } S_{11}^* - S_{12}^* = 0$$

$$\Rightarrow S_{11}^* = S_{12}^*$$

$$\text{ie, } S_{11} = S_{12} \quad \text{--- (9)}$$

Equating equations (6) & (7)

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = |S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2$$

$$|S_{11}|^2 = |S_{22}|^2$$

$$\Rightarrow S_{11} = S_{22} \quad \text{--- (10)}$$

Sub. eq(8) & (9) in equation (6)

$$2|S_{12}|^2 + \frac{1}{2} = 1$$

$$|S_{12}|^2 = \frac{1}{4}$$

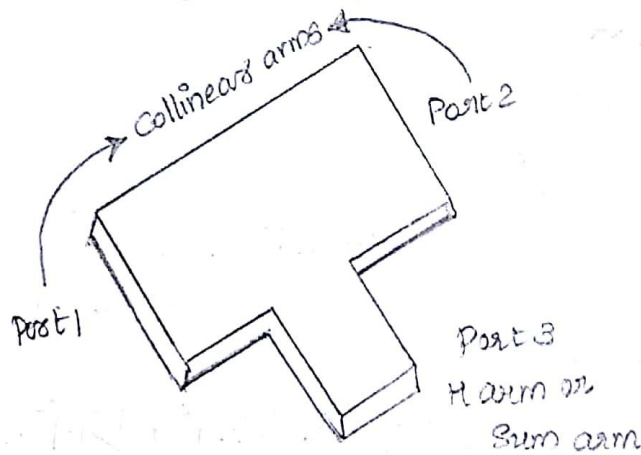
$$S_{12} = \frac{1}{2}$$

∴ the scattering matrix of E-plane Tee is

$$\therefore S = \begin{bmatrix} 1/2 & 1/2 & 1/\sqrt{2} \\ 1/2 & 1/2 & -1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \end{bmatrix}$$

H Plane Tee

In H plane Tee the side arm or H arm is parallel to the magnetic field. The signal fed to one of the ports will be divided between the other two ports and the signals will be in phase. The o/p of the H plane Tee is the sum of i/p signals.



The general matrix, $S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$ — (1)

Since, the S_g signals are in phase, $S_{13} = S_{23}$ — (2)

If port 3, is matched $S_{33} = 0$. — (3)

By Symmetry

$$\left. \begin{array}{l} S_{12} = S_{21} \\ S_{13} = S_{31} \\ S_{23} = S_{32} \end{array} \right\} \text{--- (4)}$$

applying eqn (2), (3) & (4) in (1)

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{13} \\ S_{13} & S_{13} & 0 \end{bmatrix} \quad \text{--- (5)}$$

by unitary property $[S][S]^* = I$.

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{13} \\ S_{13} & S_{13} & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{12}^* & S_{22}^* & S_{13}^* \\ S_{13}^* & S_{13}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_{11} \Rightarrow |S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1 \quad \text{--- (6)}$$

$$R_{22} \Rightarrow |S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 = 1 \quad \text{--- (7)}$$

$$R_{33} \Rightarrow |S_{13}|^2 + |S_{13}|^2 = 1$$

$$2|S_{13}|^2 = 1$$

$$S_{13} = \frac{1}{\sqrt{2}} \quad \text{--- (8)}$$

$$R_{12} \Rightarrow S_{13} S_{11}^* + S_{13} S_{12}^* = 0.$$

$$S_{13} [S_{11}^* + S_{12}^*] = 0$$

$$\rightarrow S_{11}^* = -S_{12}^*$$

$$S_{11} = -S_{12} \quad \text{--- (9)}$$

equating eqn (6) & (7)

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = |S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2$$

$$|S_{11}|^2 = |S_{22}|^2$$

$$S_{11} = S_{22} \quad \text{--- (10)}$$

Sub eq (8) & (9) in eqt (6)

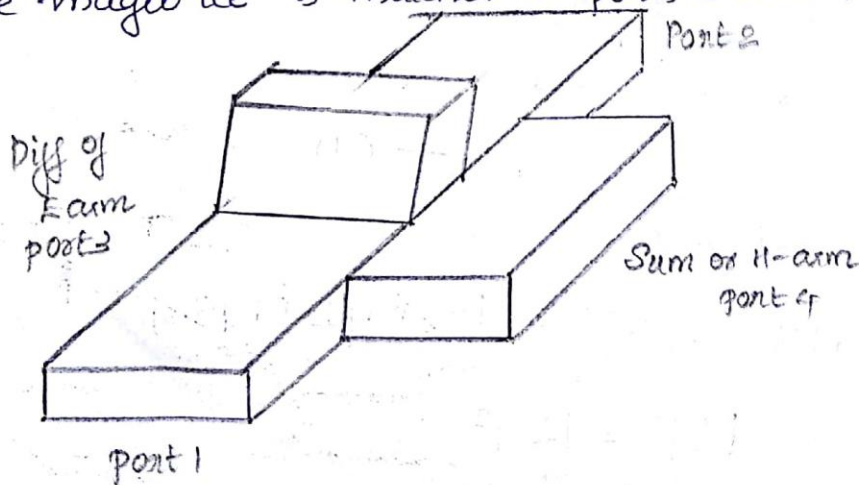
$$\frac{1}{\sqrt{2}}$$

$$S = \begin{bmatrix} \gamma_2 & -\gamma_2 & \gamma_2 \\ -\gamma_2 & \gamma_2 & \gamma_2\sqrt{2} \\ \gamma_2\sqrt{2} & \gamma_2\sqrt{2} & 0 \end{bmatrix}$$

11/01/18
Monday

Hybrid Tee or Magic Tee

A combination of E plane Tee and H plane Tee is called hybrid Tee or magic tee. It consists of four ports, if two waves of equal magnitude and same phase are fed into port 1 and port 2, the o/p will be subtractive and zero at port 3 and will be additive at port 4. A wave incident at port 4 divides equally between port 1 and 2, and will not appear at port 3. A wave incident at port 3 will produce an o/p of equal magnitude and opposite phase at ports 1 & 2. The magic tee is matched at ports 3 and 4.



The general matrix of the magic Tee is given by

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \quad \text{--- (1)} \quad "S_{42} = S_{24}"$$

From the property of symmetry $S_{14} = S_{41}$, $S_{13} = S_{31}$, $S_{23} = S_{32}$ --- (2)

Since port 3 acts as the E plane Tee

$$S_{13} = -S_{23} \quad \text{--- (3)}$$

Since port 4 acts as a H plane Tee, $S_{14} = S_{24}$ --- (4)

Considering the phase delay in the lines

$$\left. \begin{aligned} S_{34} = S_{43} &= 0 \\ S_{12} = S_{21} &= 0 \end{aligned} \right\} \quad \text{--- (5)}$$

If port 3 and port 4 are matched

$$S_{33} = S_{44} = 0 \quad \text{--- (6)}$$

applying eq (2) to (6) in eqⁿ (1)

$$S = \begin{bmatrix} S_{11} & 0 & S_{13} & S_{14} \\ 0 & -S_{22} & -S_{13} & S_{14} \\ S_{13} & -S_{13} & 0 & 0 \\ S_{14} & S_{14} & 0 & 0 \end{bmatrix} \quad \text{--- (7)}$$

By unitary property matrix $[S][S^*] = I$

$$\begin{bmatrix} S_{11} & 0 & S_{13} & S_{14} \\ 0 & -S_{22} & -S_{13} & S_{14} \\ S_{13} & -S_{13} & 0 & 0 \\ S_{14} & S_{14} & 0 & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & 0 & S_{13}^* & S_{14}^* \\ 0 & -S_{22}^* & -S_{13}^* & S_{14}^* \\ S_{13}^* & -S_{13}^* & 0 & 0 \\ S_{14}^* & S_{14}^* & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_1 C_1 = |S_{11}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1 \quad \text{--- (8)}$$

$$R_2 C_2 = |S_{22}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1 \quad \text{--- (9)}$$

$$R_3 C_3 = 2|S_{13}|^2 = 1 \Rightarrow S_{13} = 1/\sqrt{2} \quad \text{--- (10)}$$

$$R_4 C_4 = 2 |S_{14}|^2 = 1$$

$$\Rightarrow \underline{S_{14}} = \frac{1}{\sqrt{2}} \quad \text{--- (11)}$$

Sub. eqn (10) & (11) in eqn (8)

$$|S_{11}|^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 = 1$$

$$|S_{11}|^2 = 1 - 1$$

$$\Rightarrow \underline{S_{11}} = 0$$

Equating eqn (8) & (9)

$$|S_{11}|^2 + |S_{13}|^2 + |S_{14}|^2 = |S_{22}|^2 + |S_{13}|^2 + |S_{14}|^2$$

$$\Rightarrow \underline{S_{11}} = S_{22}$$

∴ the matrix of magic Tee is

$$S = \begin{bmatrix} 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \end{bmatrix}$$

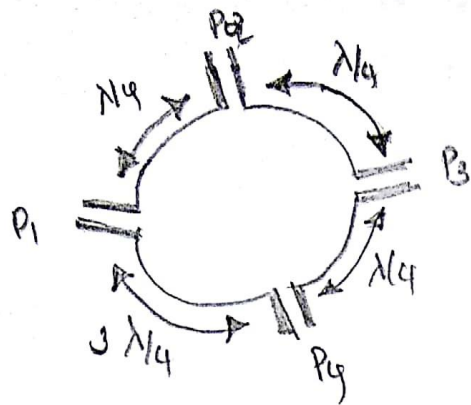
Hybrid Rings

Hybrid ring circuits are also known as 'rat race coupler'.

These junctions overcome the power limitations of magic tee.

It is constructed by folding rectangular waveguides into circular waveguides. This junction has 4 ports with upper 3 ports separated by $\frac{\lambda}{4}$ and lower two ports separated by $\frac{3\lambda}{4}$. when a wave is fed into port 1. It will not appear at port 2.

at port 3 due to the phase shifts: Similarly waves fed onto port 2 will not appear at port 4 due to phase difference



The general matrix of hybrid ring is

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \quad \text{--- (1)}$$

If the ports 1, 2, 3 & 4 are matched then, $S_{11} = S_{22} = S_{33} = S_{44} = 0$ --- (2)

Considering the i/p-o/p conditions

$$S_{13} = S_{31} = 0 \quad \text{and} \quad S_{24} = S_{42} = 0$$

$$S_{21} = -S_{41}$$

∴ the general matrix can be written as

$$S = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} - S_{12} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ -S_{12} & 0 & S_{34} & 0 \end{bmatrix}$$

$$[S][S]^* = I$$

$$\begin{bmatrix} 0 & S_{12} & 0 & -S_{12} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ -S_{12} & 0 & S_{34} & 0 \end{bmatrix} \begin{bmatrix} 0 & S_{12}^* & 0 & -S_{12}^* \\ S_{12}^* & 0 & S_{23}^* & 0 \\ 0 & S_{23}^* & 0 & S_{34}^* \\ -S_{12}^* & 0 & S_{34}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_1 C_1 = |S_{12}|^2 + |S_{12}|^2 = 1$$

$$2|S_{12}|^2 = 1$$

$$\Rightarrow |S_{12}|^2 = \frac{1}{2}$$

$$\underline{\underline{S_{12} = 1/\sqrt{2}}}$$

$$R_2 C_2 = |S_{12}|^2 + |S_{23}|^2 = 1$$

$$\frac{1}{2} + |S_{23}|^2 = 1$$

$$|S_{23}|^2 = \frac{1}{2}$$

$$\underline{\underline{S_{23} = 1/\sqrt{2}}}$$

$$R_3 C_3 = |S_{23}|^2 + |S_{34}|^2 = 1$$

$$\frac{1}{2} + |S_{34}|^2 = 1$$

$$\underline{\underline{S_{34} = 1/\sqrt{2}}}$$

$$R_4 C_4 = |S_{12}|^2 + |S_{34}|^2 = 1$$

$$S_{12} = 1/\sqrt{2}$$

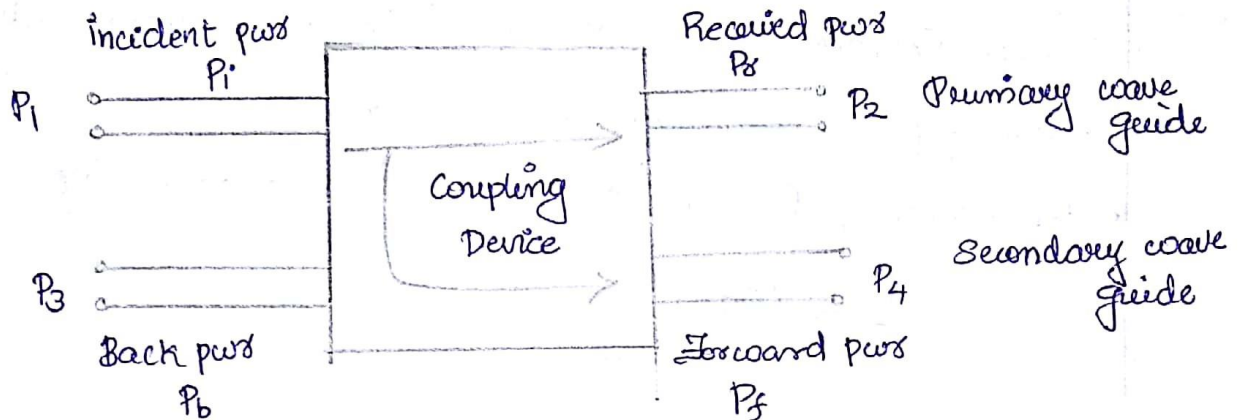
\therefore matrix of Hybrid Rings is

$$\underline{\underline{S = \begin{bmatrix} 0 & 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 0 & 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \end{bmatrix}}}$$

Directional Coupler

3/10/18

Directional coupler is a 4 port device which has primary & secondary waveguides. The primary waveguide is from port 1 to port 2. & secondary waveguide is from port 3 to port 4.



→ Directional Coupler is used to couple microwave power which is unidirectional in most of the cases. The properties of a directional coupler are

- i) All the ports are matched.
- ii) when the power travels from port 1 to port 2 some portion of it gets coupled to port 4 but not to port 3.
- iii) when the power travels from port 2 to port 1 some portion of it gets coupled to port 3 but not to port 4.
- iv) The coupling factor of a directional coupler is the ratio of incident power to forward power.

$$\text{Coupling factor} = 10 \log \frac{P_i}{P_f}$$

→ Directivity of the directional coupler is the ratio of forward power to back power

$$\text{Directivity} = 10 \log \frac{P_f}{P_b}$$

→ Isolation of a directional coupler is the ratio of incident power to back power.

$$I = 10 \log \frac{P_i}{P_b}$$

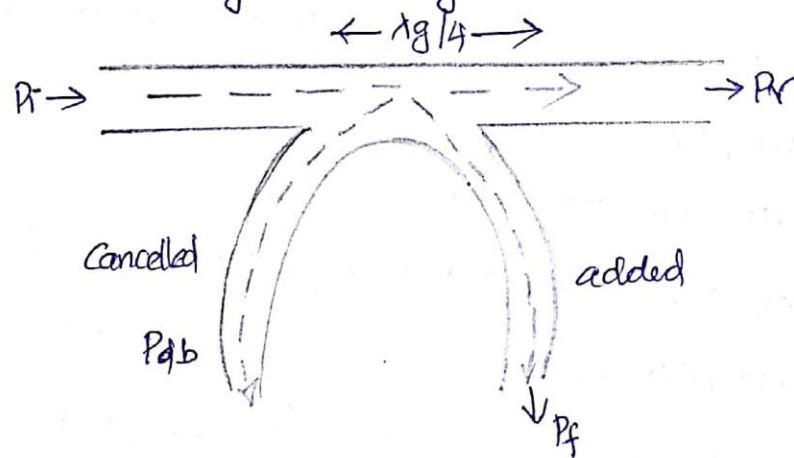
→ Isolation = Coupling factor + Directivity.

→ Two hole directional coupler is same as conventional directional coupler but with two holes in common between primary & secondary waveguides.

→ The spacing between these two holes is given by

$$L = (2n+1) \frac{\lambda_g}{4}$$

where, n = an integer
 λ_g - wavelength



→ A fraction of energy entering into port 1 passes through the holes and is radiated into port 2. The forward waves in port 4 are in the same phase and are added.

→ The backward waves in port 3 are out of phase and are cancelled.

The general S matrix of a directional coupler is

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \quad \text{--- (1)}$$

→ Since all ports in a directional coupler are matched.

$$S_{11} = S_{22} = S_{33} = S_{44} = 0. \quad \text{--- (2)}$$

→ Since there is no coupling between ports 1 & 3. and port 2 & 4

$$S_{13} = S_{31} = S_{24} = S_{42} = 0. \quad \text{--- (3)}$$

Apply eqn (2) & (3) in (1).

$$S = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix}$$

By unitary property

$$[S][S]^* = I$$

$$\begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix} \begin{bmatrix} 0 & S_{12}^* & 0 & S_{14}^* \\ S_{12}^* & 0 & S_{23}^* & 0 \\ 0 & S_{23}^* & 0 & S_{34}^* \\ S_{14}^* & 0 & S_{34}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_1 C_1 = |S_{12}|^2 + |S_{14}|^2 = 1 \quad \text{--- (4)}$$

$$R_2 C_2 = |S_{12}|^2 + |S_{23}|^2 = 1 \quad \text{--- (5)}$$

$$R_3 C_3 = |S_{23}|^2 + |S_{34}|^2 = 1 \quad \text{--- (6)}$$

$$R_4 C_4 = |S_{14}|^2 + |S_{34}|^2 = 1 \quad \text{--- (7)}$$

$$R_{11}C_3 = S_{12} S_{23}^* + S_{14} S_{34}^* = 0 \quad \text{--- (7)}$$

comparing eq (4) & (5)

$$|S_{12}|^2 + |S_{14}|^2 = |S_{12}|^2 + |S_{23}|^2$$

$$\underline{S_{14} = S_{23}} \quad \text{--- (8)}$$

Comparing eq (5) & (6)

$$|S_{12}|^2 + |S_{23}|^2 = |S_{34}|^2 + |S_{23}|^2$$

$$\underline{S_{12} = S_{34}} \quad \text{--- (9)}$$

Let, S_{12} be real and +ve

$$\text{ie, } S_{12} = S_{34} = P \quad \text{--- (10)}$$

applying eqⁿ (10) in eqⁿ (7)

$$\therefore P S_{23}^* + S_{14} P = 0$$

$$P [S_{23}^* + S_{14}] = 0$$

$$P [S_{23}^* + S_{23}] = 0$$

from eqⁿ (8)

$$S_{23}^* + S_{23} = 0$$

$$\therefore S_{23} = -S_{23}^*$$

To satisfy the above condition S_{23} should be a Complex Value.

$$\text{Let, } S_{23} = jQ$$

\therefore The S matrix of directional coupler is.

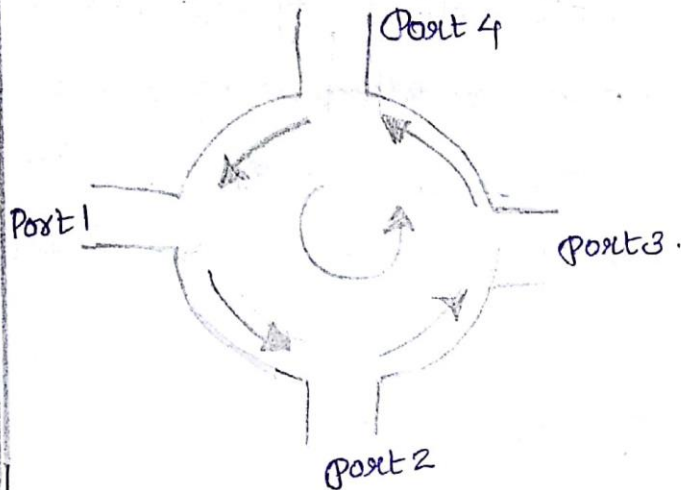
$$S = \begin{bmatrix} 0 & P & 0 & S_{14} jQ \\ jQ & 0 & jQ & 0 \\ 0 & jQ & 0 & P \\ jQ & 0 & P & 0 \end{bmatrix}$$

Q write notes on hybrid corners, bends & twists.

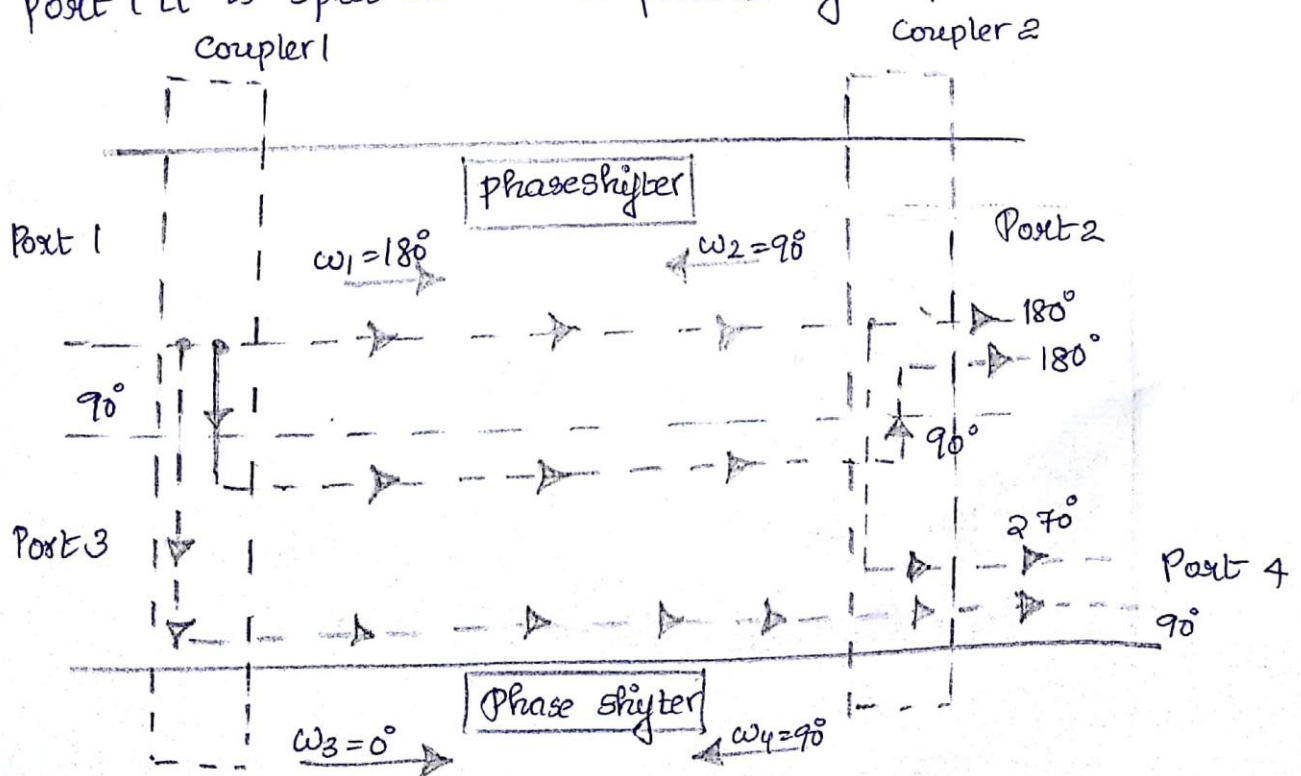
Microwave Circulators

A microwave circulator is a multiport waveguide in which the waves can flow only from 1 port to the next immediate port only in one direction.

i.e., the waves travels from n^{th} port to $(n+1)^{\text{th}}$ port.



Each of the couplers in the circulator introduces a phase shift of 90° . Each of the phase shifters produces certain amount of phase change in the direction indicated. when a wave is incident on port 1 it is split into 2 components by coupler 1.



The wave in the primary waveguide arrives at port 2 with a phase change of 180° . The 2nd wave propagates through 2 couplers and the 2^o wave guide & arrives at port 2 with a phase change of 180° . Since the 2 waves reaching port 2 are in phase. Power is transferred from port 1 to port 2. The wave propagates through primary waveguide phase shifter and coupled 2 & arrives at port 4 with a phase change of 270° . The wave travels through coupler 1 and secondary waveguide and arrives at port 4 with a phase shift of 90° . Since the waves reaching port 4 are out of phase the power transmission from port 1 to port 4 is zero.

The general S matrix of circulator is

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \quad \text{--- (1)}$$

Since all ports in a circulator are matched,

$$S_{11} = S_{22} = S_{33} = S_{44} = 0 \quad \text{--- (2)}$$

$$\left. \begin{aligned} S_{12} &= S_{13} = 0 \\ S_{32} &= S_{24} = 0 \\ S_{31} &= S_{34} = 0 \\ S_{41} &= S_{42} = 0 \end{aligned} \right\} \quad \text{--- (3)}$$

$$S_{14} = S_{21} = S_{32} = S_{43} = 1 \quad \text{--- (4)}$$

$$S = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Microwave Isolator.

An isolator is a device that is used to isolate one component from the reflections of other components in the transmission line. An ideal isolator provides complete signal power transmission in one direction and complete signal attenuation in opposite direction.

Isolators are also known as unidirectional devices. They are used to improve the frequency stability of klystrons & magnetrons. The commonly used type of isolator is 'Faraday Rotation isolator'. This type of isolator transmits the waves which are \perp to the resistive plane in the input and reflect the wave which are parallel to the resistive plane in the output. The o/p resistive part is placed 45° with respect to input resistive plane. The degree of rotation depends on the length & diameter of the ferrite rod and the applied DC magnetic field. The wave incident on input resistive plane is applied to the i/p resistive plane by the principle of perpendicular line of transmission.

These waves are transmitted by the ferrite rod towards the output resistive plane. Thus, transmission wave occurs from input to output.

The reflected wave from the o/p end is rotated clockwise 45° by the ferrite rod. Thus the reflected wave made parallel to resistive plane and is absorbed by resistive plane rather than transmitting.

MODULE V

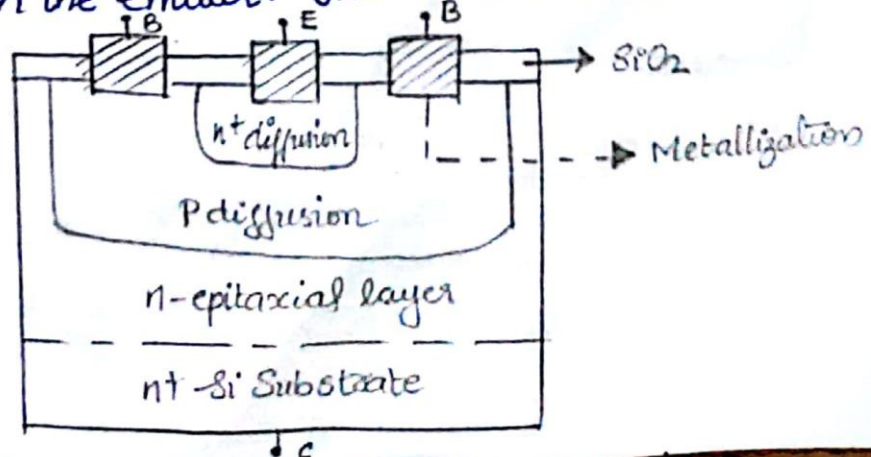
SOLID STATE MICROWAVE DEVICES

MICROWAVE BIPOLAR TRANSISTOR

The microwave bipolar transistor is a nonlinear device which is mostly silicon npn type operating upto 50GHz . The Geometry of the transistor can be characterised as interdigitated geometry, over lay geometry and matrix geometry. [These geometries have aside emitter area to overcome transit time limitations]. The interdigitated geometry is used for small signal, small power circuits. The over lay and matrix types are used for small power only. For high frequency applications the NPN structure is preferred because the electron mobility is higher than hole mobility. [Diffusion and ion implantation are the common methods used for transistor fabrication.]

Basic Construction

An epitaxial n layer is grown over a low resistivity n^+ silicon substrate. Above the epitaxial layer a p region is diffused forming the base & n^+ layer is diffused over the p region to form the emitter. The silicon substrate acts as the collector.



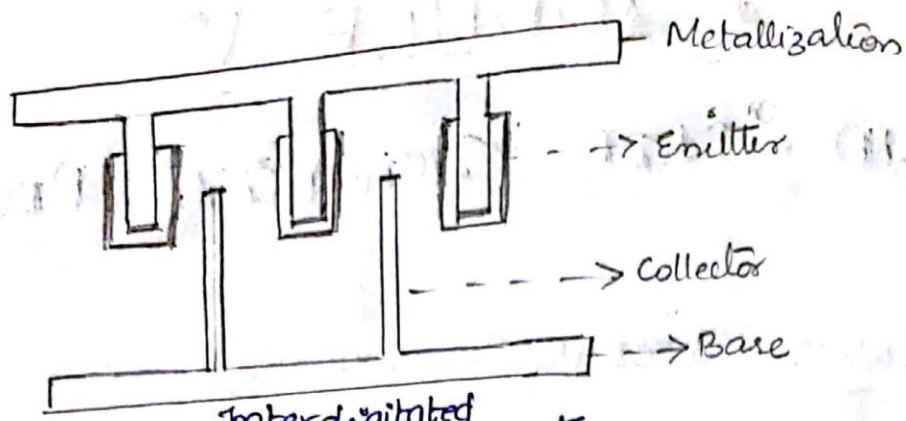


Fig: Interdigitated Integrated Geometry.

The microwave bipolar transistors are active three terminal devices which is commonly used for amplification process & switching phenomena. The three regions of the transistor are emitter, base & collector. The emitter region forms the i/p of the device and the collector region forms the o/p of the device. The emitter region of the transistor is heavily doped & has moderate area of cross section. The base of the transistor is thin and lightly doped to reduce the recombination rate. The collector region of the transistor is large and moderately doped (The charge carriers from the emitter are supplied to the collector through the base) when the charge carriers from the emitter reach the base some of them recombine with the charge carriers in the base. The remaining charge carriers are directed towards the collector constituting the collector current or i/p current.

Modes of Operation

The microwave transistors have four modes of operation depending on the polarity of the applied voltage.

1. NORMAL MODE

In this mode the emitter base junction of the npn transistor is forward biased & collector base junction is reverse biased. Most transistor amplifiers are operated in normal mode.

2. SATURATION MODE

When both the emitter base junction & collector base junction are forward biased the transistor is in saturation mode with low resistance and acts like a short circuit.

3. CUT OFF MODE

When both the T^j junctions are reverse biased, the T^r is operated in cut off mode. The T^r acts like an open ckt. Both the saturation & cut off modes are used when transistor acts as a switch.

4. INVERTED ACTIVE MODE

In this mode the emitter base junction is reverse biased & collector base junction is FB.

Power Frequency Limitations

Microwave transistors have limitations on frequency & power. These limitations can be due to maximum velocity of carriers, maximum electric field & max current. The four basic equations for the power frequency limitation are

1) Voltage - frequency limitation

$$V_m f_T = \frac{E_m V_s}{2\pi}$$

where f_T - cut off frequency

$$f_T = \frac{1}{2\pi\tau}$$

$$\tau = \frac{L}{V}$$

V_m - maximum voltage

V_s - velocity

E_m - max electric field.

when, the length decreases the average time τ decreases as a result the frequency increases. when the frequency increases the applied max voltage decreases.

ii) Current frequency Limitation

$$I_m \times C f_T = \frac{E_m V_s}{2\pi}$$

where, X_c - impedance

I_m - max current

If the impedance level is zero, the maximum current is infinite. The impedance value should be maintained in a such a way that max current is obtained for producing max power.

iii) Power - frequency Limitation.

$$\sqrt{P_m X_c} f_T = \frac{E_m V_s}{2\pi}$$

If the value of X_c is zero, the maximum power delivered is infinite.

iv) Power Gain frequency Limitation

$$\sqrt{G_m V_m V_{th}} f_T = \frac{E_m V_s}{2\pi}$$

G_m - max gain

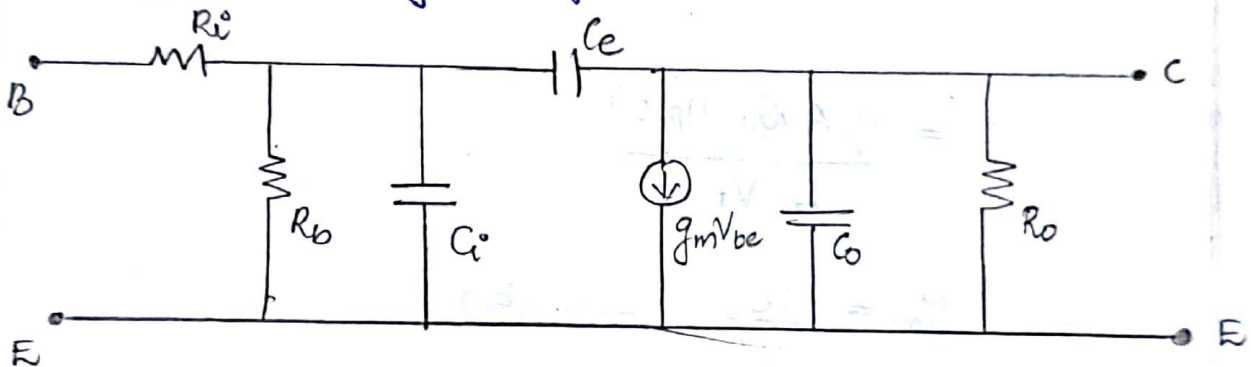
V_{th} - thermal Voltage.

V_m - maximum Voltage.

If the frequency increases, the gain of the device decreases.

Equivalent Model of Microwave Bipolar Transistor

Hybrid π equivalent model is Commonly used in normal active mode for Small Signal operations



A change of emitter Voltage V_{be} at the i/p terminal will induce a change of collector current at the o/p terminal. The mutual conductance or transconductance for the small signal model is given by

$$g_m = \frac{\partial i_c}{\partial V_{be}} \quad \text{--- (1)}$$

The density of charge carriers across the junction is given by

$$n_p(\omega) = n_{p0} e^{V_{be}/V_T} \quad \text{--- (2)}$$

The collector current is defined based on the charge density as

$$i_c = \frac{q A D_n n_p(\omega)}{L_n} \quad \text{--- (3)}$$

where, q - charge

A - area of cross section

D_n - diffusion constant

$n_p(\omega)$ - charge density L_n - length.

Sub eq (2) in eqⁿ (3)

$$i_c = \frac{q A D_n n_{p0} e^{V_{be}/V_T}}{L_n} \quad \text{--- (4)}$$

$$\therefore g_m = \frac{\partial i_c}{\partial V_{be}}$$

$$g_m = \frac{q A D_n n_{p0} e^{V_{be}/V_T}}{L_n V_T}$$

$$= \frac{q A D_n n_p(\omega)}{L_n V_T}$$

$$\boxed{g_m = \frac{i_c}{V_T}} \quad \text{--- (5)}$$

diffusion capacitance,

$$C_{be} = \frac{dQ_b}{dV_{be}}$$

where, Q_b - charge stored in the base.

$$Q_b = \frac{q n_p(\omega) \omega_b A}{2}$$

Sub. for $n_p(\omega)$ from (2)

$$Q_b = \frac{q n_{p0} e^{V_{be}/V_T} \omega_b A}{2}$$

$$\therefore C_{be} = \frac{q \omega_b A n_{p0} e^{V_{be}/V_T}}{2 V_T}$$

$$C_{be} = \frac{q \omega_b A n_p(\omega)}{2 V_T}$$

$$C_{be} = \frac{i_c L_n \omega_b}{2 V_T D_n}$$

from eqn 3.

from eqn (5)

$$\text{i.e., } C_{be} = \frac{g_m L_n \omega_b}{2 D_n}$$

$$\text{If, } L_n = \omega_b$$

$$C_{be} = \frac{g_m \omega_b^2}{2 D_n}$$

The i/p conductance,

$$g_b = \frac{i_c}{h_{fe} V_T}$$

$$g_b = \frac{g_m}{h_{fe}}$$

where, h_{fe} - i/p resistance / Forward emitter resistance,

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Tuesday

Tunnel Diodes

The tunnel diode is a PN junction device, that operates in certain regions of V-I characteristics by the tunneling of electrons across the potential barrier of the junction. This device can be used in high speed switching and logic circuits. Tunnel diodes are useful in many applications such as microwave amplifiers, microwave oscillators etc because of its low cost, light weight, high speed, low power, low noise and high peak current to valley current ratio.

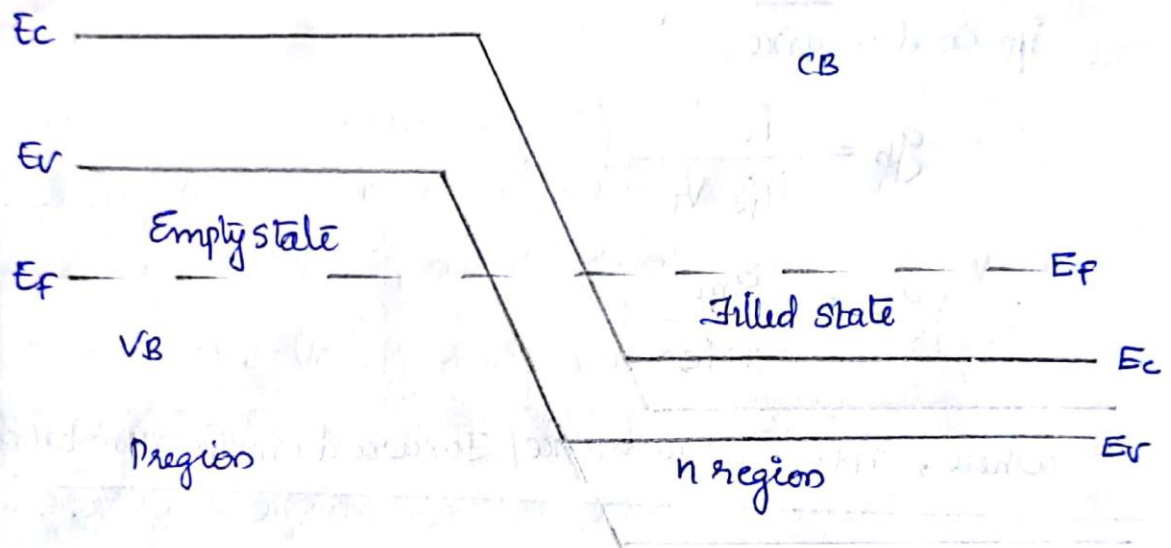
Characteristics.

Principle of operation

i) Under Equilibrium

The Fermi level is constant throughout the junction, the Fermi level lies below the valence band in the P side & above the conduction

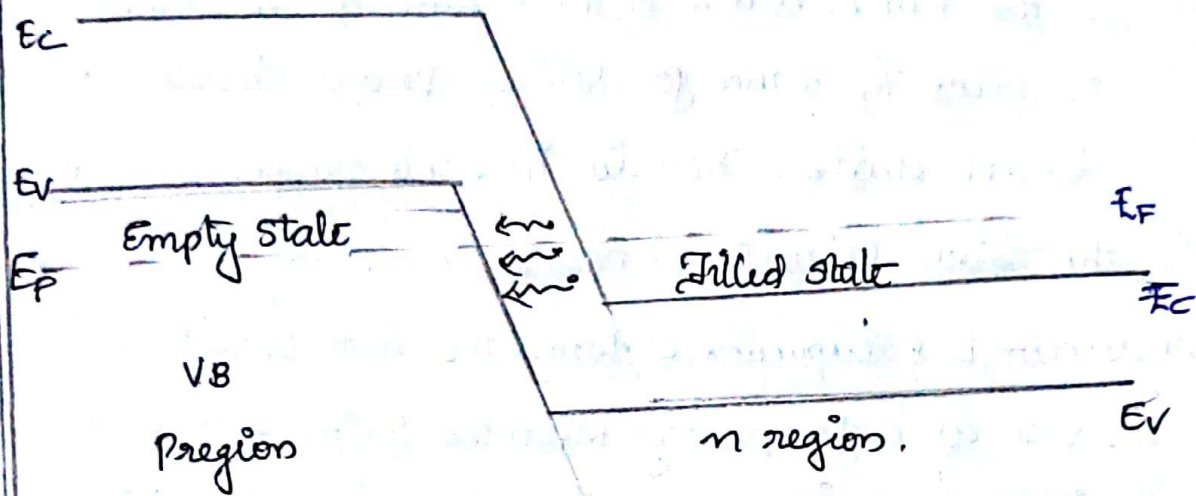
band in 'n' side. Since there are no filled states on one side of the junction which is at the same energy level as the empty state on the other side no flow of charge occurs in either directions & the current is zero. [The bands must overlap for the Fermi level to be constant]. With a small FB or RB, filled and empty states appear on opposite sides separated by width of the depletion region.



ii) Under Forward Bias

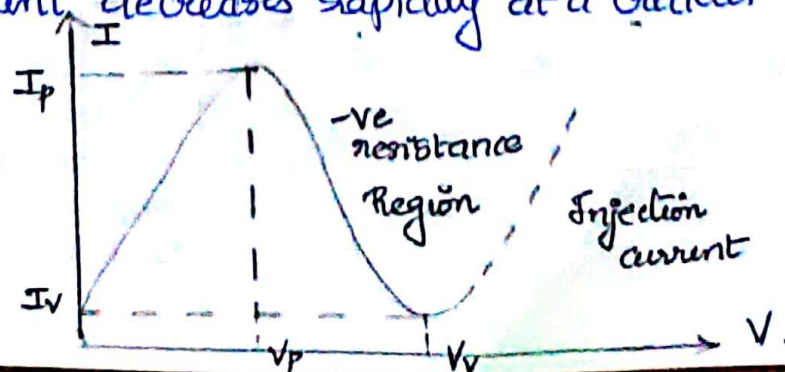
When the FB is applied across the junction the potential barrier is decreased by the magnitude of applied voltage. A difference in Fermi levels is created in both sides. Since, there are filled states in the conduction band of 'n' region at the same energy level as the allowed empty states in the valence band of p region electrons tunnel through the barrier from n region to p region giving rise to forward tunneling current from p region to n region. As the FB is increased to a max voltage a max no: of electrons can tunnel through the barrier producing peak current. If the voltage is increased further, the tunneling current decreases as there are no empty states available in the p region. When the FB voltage is

increased further, the current flow increases which is mainly due to minority charge carriers and is known as 'injection current'.



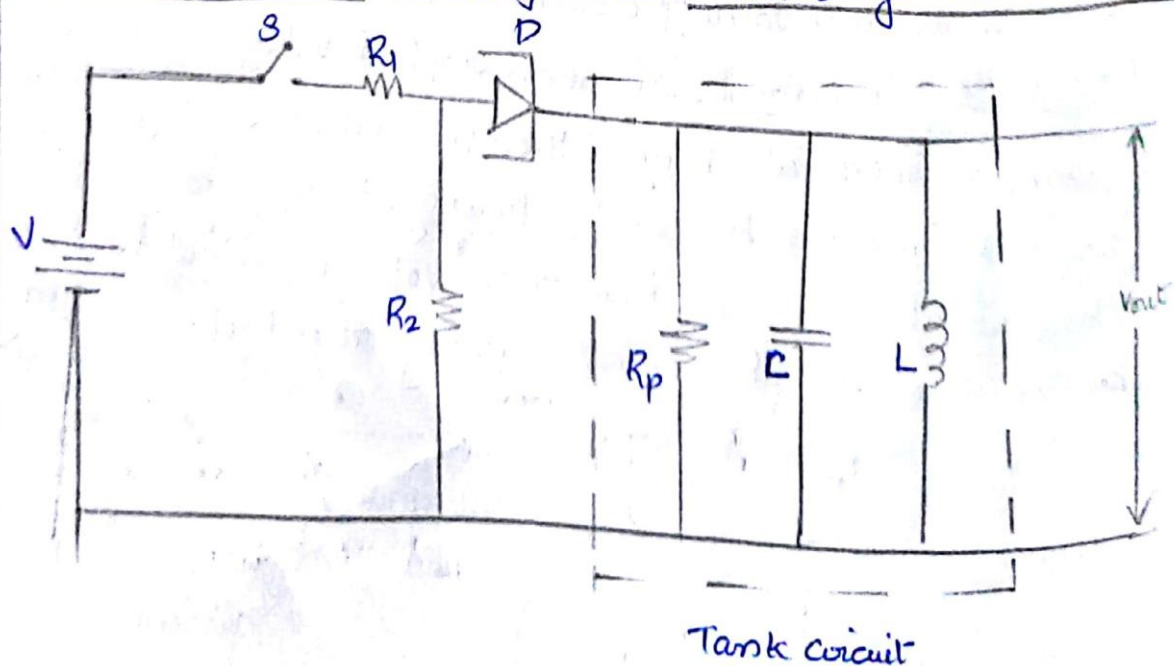
Tunnel Diode Characteristics

When the applied forward voltage is betⁿ 0 and V_p , the electrons tunnel from n region to p region, thereby increasing the current as the applied forward voltage reaches a value V_p , the current flowing across the junction attains a max value called the 'Peak Current' I_p . When the applied voltage is increased further a decrease of current occurs this produces a region of -ve slope. The maximum value of current achieved in -ve resistance region is called 'Valley current', I_v at an applied valley voltage V_v . If the voltage is increased beyond the -ve resistance region the current begins to increase due to the flow of minority charge carriers. This characteristic is also known as 'Voltage Controlled -ve resistance'. As the current decreases rapidly at a critical voltage.



Tunnel Diode Oscillator

The value of resistor should be in such a way that it biases the tunnel diode in the middle of -ve resistance region. The resistor R_1 is used for setting proper biasing point for the tunnel diode. Resistor R_2 sets proper biasing point for the tank circuit. A parallel combination of resistor R_p , inductor, L & capacitor, C forms the tank circuit which resonates at a frequency. When the switch 'S' is closed the circuit current increases whose value is determined by the value of resistor R_1 and the diode resistance. [Voltage drop across the tunnel diode increases as the applied voltage increases] As a result the tunnel diode is driven into -ve resistance region. In this region current starts decreasing until the voltage across the diode equals the valley voltage. At this point further increase of the applied voltage increases current. This increase in current will increase the voltage drop across the resistor which produces the voltage across the diode.

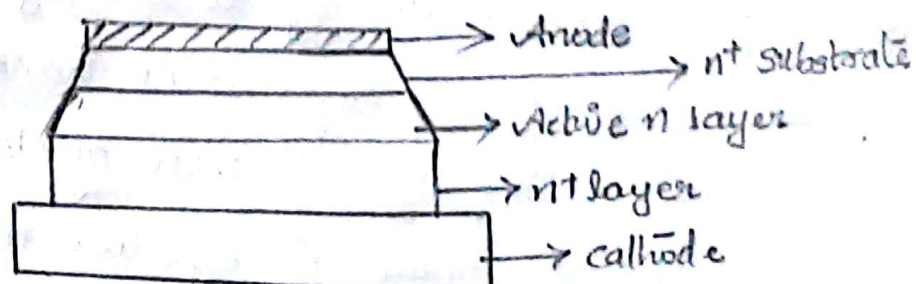


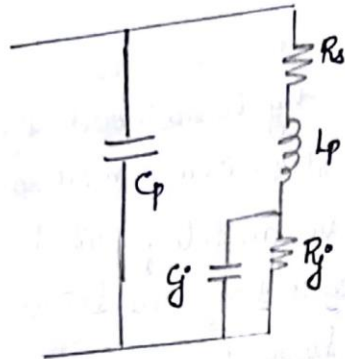
Gunn Diode.

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Microwave devices that operate by transferred electron mechanism are called Gunn diodes. Some materials like GaAs shows the behaviour of -ve mobility with increase in electric field. This phenomenon is caused by the transfer of conduction band electrons from lower energy high mobility state to high energy low mobility state. This is known as 'transferred electron effect' or 'Ridley Watkins effect' or 'Hilsum' (RWH)'.

The devices based on this effect are called transferred electron devices. Gunn diodes are -ve resistance devices which are normally used as low power oscillators at microwave frequencies. The basic structure of Gunn diode consists of n type GaAs semi-conductor. Heavily doped n⁺ regions are formed over the substrate. If the voltage or electric field is applied to GaAs initially the current will increase with voltage. When the voltage exceeds the threshold voltage a high electric field is produced and the electrons are excited from initial lower state to higher state. If the rate at which the electrons transferred is high the current will decrease with increase in voltage producing -ve resistance.





G - Diode Capacitance

R_j - Diode resistance

R_s - total resistance

C_p - package capacitance.

L_p - Package inductance.

→ The Conduction band has two valleys

i) Central valley with low energy & high mobility.

ii) Satellite valley with high energy & low mobility.

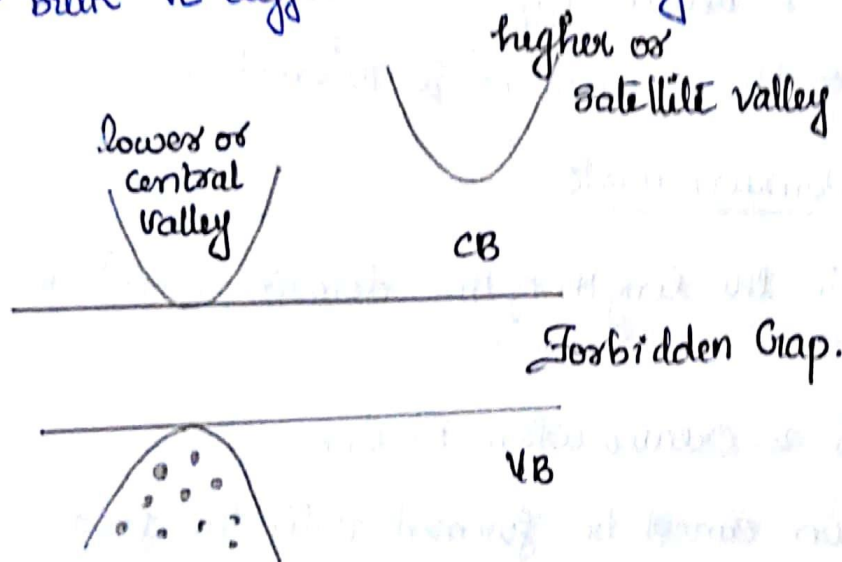
→ Under normal conditions electrons are in the central valley. When the electric field increases the velocity of electrons increases. This happens only till the energy reaches the threshold value. Above the threshold value the mobility is non linear.

→ When the electric field increases beyond the threshold value the velocity of electrons in the central valley increases and gain enough energy to transfer to satellite valley.

Such a transfer is defined as transferred electron mechanism. The effective mass of electrons in the satellite valley is higher than the effective mass of electron in the central valley. This results in decreased mobility of

electrons in the satellite valley. Since the mobility is decreased Velocity decreases. Current density, $J = qnE$
 $= qV$.

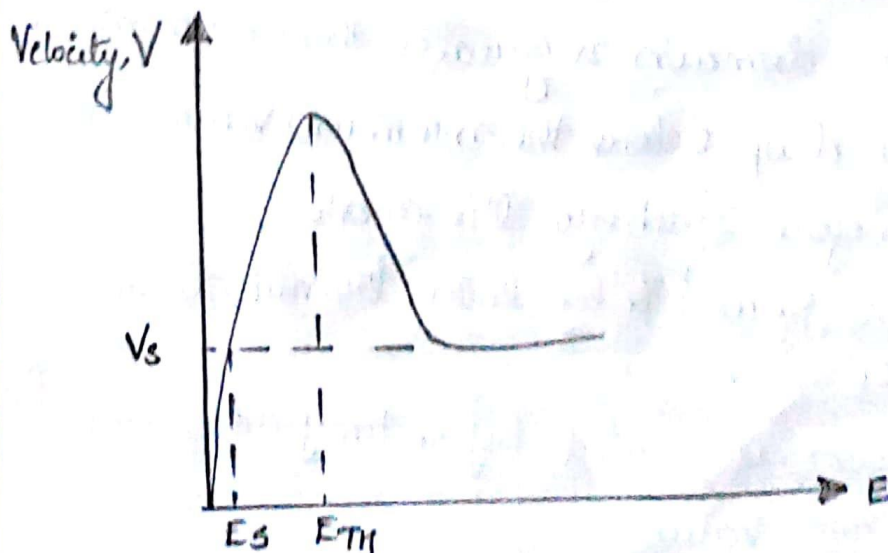
→ when velocity decreases current density decreases. Thus, when the field becomes more than the critical value the reduction in current indicate -ve resistance. This is also known as 've differential mobility' or 'bulk -ve differential conductivity' or 'Gunn effect' or 'RWH theory'.



Modes Of Operation.

1) Gunn Oscillation Mode.

This mode is defined in the regions where the product of frequency and length is about 10^7 cm/sec and the product of doping & length is greater than 10^{12} cm/sec. In normal conditions Gunn diode is operated in this mode with $E > E_{TH}$.



The three possible modes of Gunn Oscillation are

i) Transit Time Domain Mode

- Applied Velocity is equal to drift velocity
- Oscillation period is equal to transit time i.e.,
i.e., $\tau_o = \tau_t$
- Efficiency is below 10%.
- Length of the domain is 10^7 cm/sec .

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Tuesday

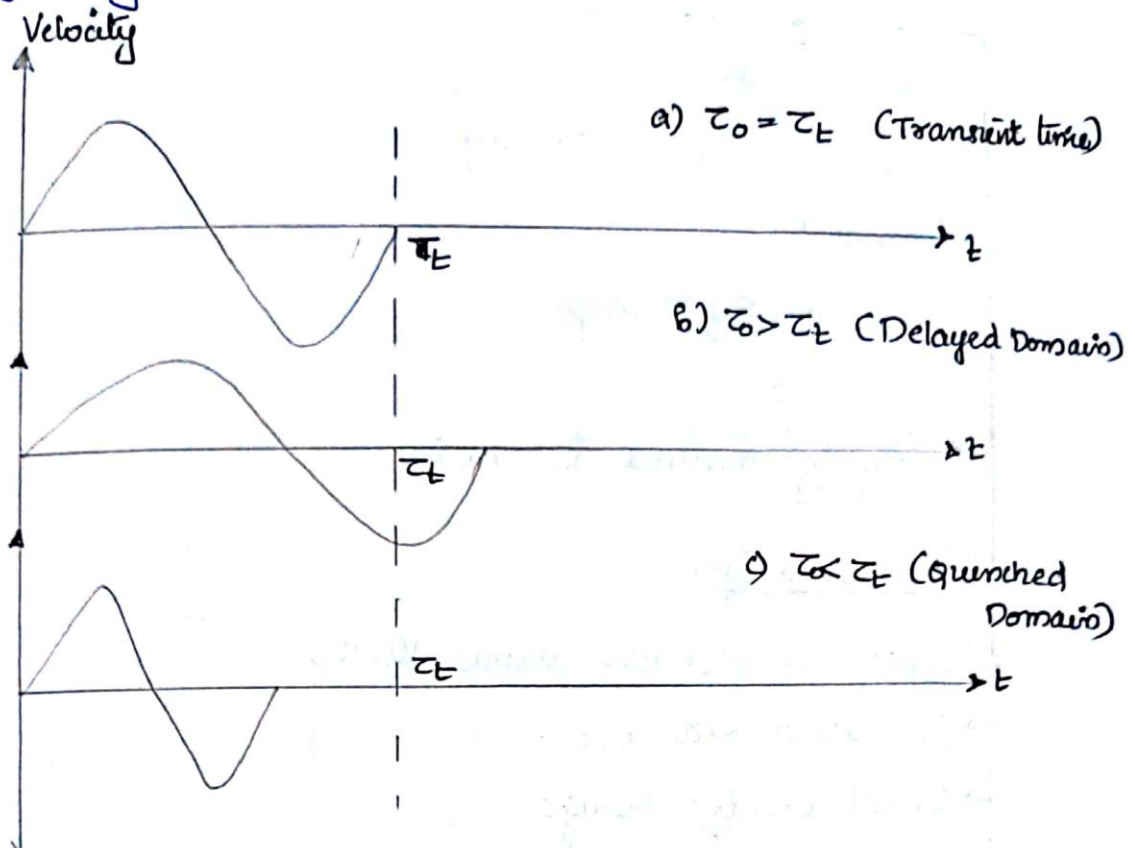
ii) Delayed Domain mode

- In this mode the length of the domain is betⁿ 10^6 cm/sec & 10^7 cm/sec .
- The domain is created when $E < E_{TH}$
- New domains cannot be formed until the field rises above the threshold.
- Oscillation period is greater than transit time.
i.e., $\tau_o > \tau_t$
- It is also known as inhibited mode.
- Efficiency is about 20%

iii) Quenched Domain Mode

- Length of the domain is greater than $2 \times 10^7 \text{ cm/sec}$
- If the field drops below the minimum value the domain collapses before reaching the anode.
- Oscillation period is less than transit time.
i.e., $\tau_o < \tau_t$.
- New domain is created before the field swings back above the min value.

→ Efficiency is about 13%.



2) Limited Space Charge Accumulation Mode

→ When the freq is very high the domains do not have sufficient time to form while the applied electric field is above the threshold value. As a result the charge carriers accumulate near the cathode & collapse with time.

→ Efficiency is about 20%

→ Oscillation time, $\tau_0 = 3 \times \tau_t$.

Gunn Diode Oscillator

One of the main applications of Gunn diode is a Gunn diode oscillator. It is used to generate & control microwave frequencies. They are mainly applied in relays, radars etc. When the Gunn diode is biased in -ve resistance region it will produce oscillations these oscillations can be in the range of GHz. The nature of oscillations depend on the diode area.

MODULE 6

Introduction to Radar Engineering

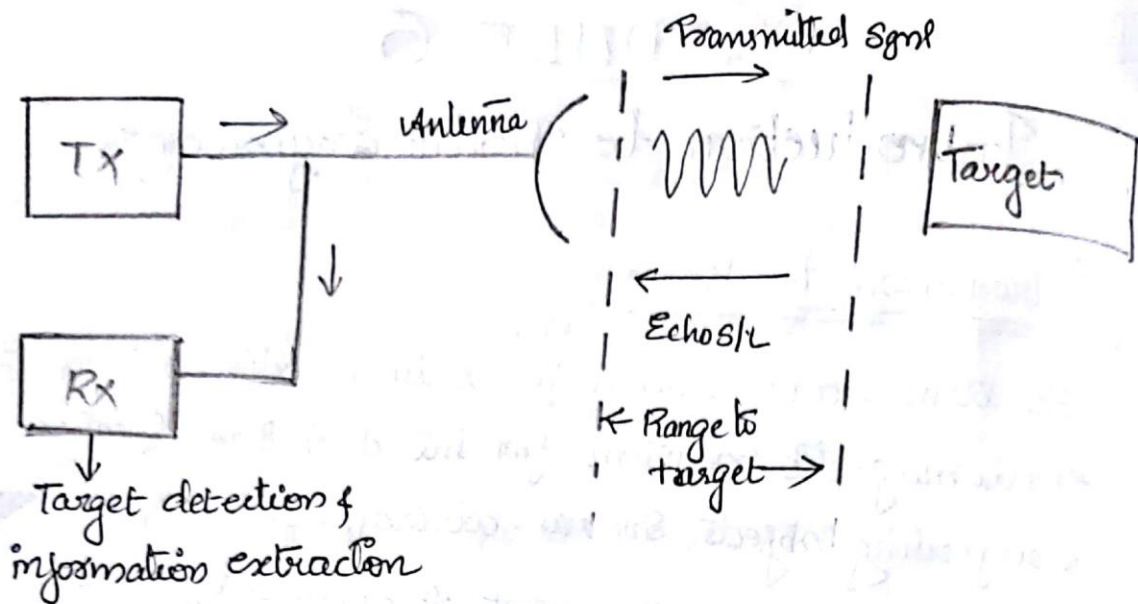
Introduction to Radar

The term radar stands for radio detection & ranging. It is an electromagnetic spectrum for the detection & location of reflecting objects such as aircraft, ships, space crafts, vehicles, people & natural environment. It operates by radiating energy into the space & detecting the echo signal reflected from the objects target. The reflected energy that is returned to the radar indicates the presence of target as well as its location. It can operate in darkness, fog, rain, snow etc. It can measure distance with high accuracy in all weather conditions.

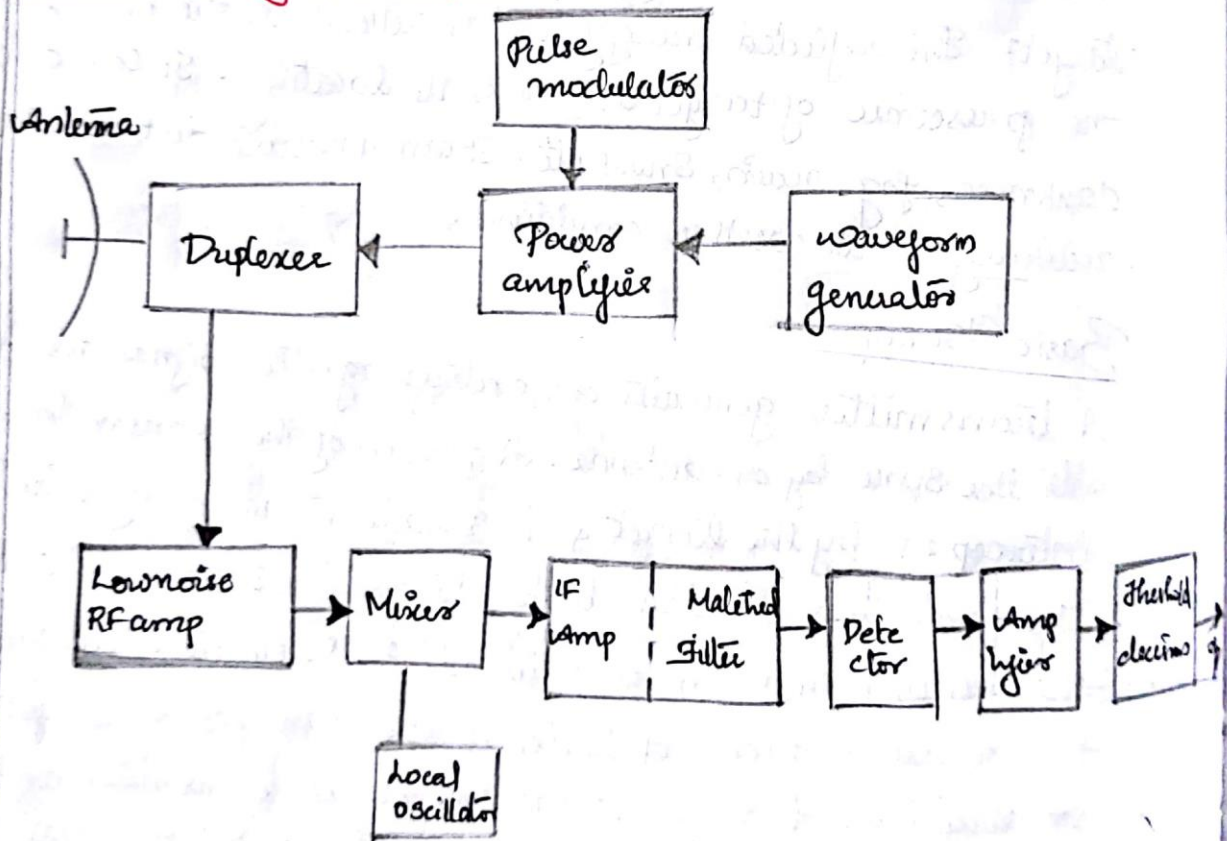
Basic Principle

A transmitter generates an electromagnetic signal that is radiated into the space by an antenna. A portion of the transmitted energy is intercepted by the target & is reradiated in many directions.

The radiations directed back towards the radar is collected by the receiving antenna & is delivered to the receiver at the receiver the signal is processed to detect the presence of target & determine its location. A single antenna can act as transmitter and receiver. The range or distance to the target is found out by measuring the time taken by the radar signal to travel to the target and return back to radar.



Block diagram of Radar



The pulse modulator can be a power amplifier such as klystron, travelling wave tube etc. It can also be a power oscillator such as magnetron. The radar signal is produced at low power by a waveform generator which is then amplified by the power amplifier. The output of the power amplifier is

delivered to the antenna by a duplexer which is then radiated into the space. The duplexer allows a single antenna to be used as both transmitter and receiver. Duplexer is a device that produces a short circuit at the i/p to the receiver when the transmitter is operating so that high power flows to the antenna & not to the receiver. On reception the duplexer directs the echo signal to the receiver & not to the transmitter. The receiver is always superhetrodyne in nature. The i/p stage is a low noise RF amplifier. The mixer and LO convert the RF signal to an intermediate frequency which is amplified by the IF amplifier. The IF amplifier is designed as a matched filter which maximizes the o/p signal to peak ratio. The matched filter maximizes the detectability of weak echo signals & attenuates unwanted signals. The IF amplifier is followed by a detector or demodulator. Its purpose is to extract the modulating signal from the carrier signal. The combination of IF amplifier and detector acts as an envelope detector which transmits the modulating signal & rejects the carrier signal. The signal at the o/p of detector is amplified by an amplifier to provide sufficient gain to the signal. At the o/p of the receiver a decision is made whether or not a target is present if the o/p is greater than the threshold the decision is that target is present. If the o/p is less than threshold it is assumed that only noise is present.

Radar Equation

The radar equation relates the range of the radar to the characteristics of transmitter, receiver, target and environment. It is useful for determining the maximum range at which a radar can detect a target. If the transmitting antenna used is isotropic in nature, the power density is given by
Power density at a range, R from an isotropic antenna is

$$P_{is} = \frac{P_t}{4\pi R^2} \quad - (1)$$

If a directive antenna of gain, G is used the power density is given by

power density at a range, R from a directive antenna is

$$P_{dir} = \frac{P_t G}{4\pi R^2} \quad - (2)$$

The radiated back power density is given by.

$P_{re} P_r$

$$\text{so radiated back power density, } P_{rad} = \frac{P_t G}{4\pi R^2} \cdot \frac{\sigma}{4\pi R^2}$$

where, σ - radar cross section

The received signal power, P_r = radiated power density \times effective area.

$$\text{i.e., } P_r = \frac{P_t G \sigma A_e}{(4\pi)^2 R^4}$$

The maximum range of radar, R_{max} is the distance beyond which the target cannot be detected. It occurs when the received signal power P_r = minimum detectable signal, S_{min} .

$$\therefore S_{min} = \frac{P_t G \sigma A_e}{(4\pi)^2 R_{max}^4}$$

$$R_{max}^4 = \left[\frac{P_t G \sigma A_e}{(4\pi)^2 S_{min}} \right]^{1/4}$$

This is the fundamental form of radar range eqⁿ.

If the same antenna is used for transmitting & receiving

The relation between gain and effective area is

$$G = \frac{4\pi A_e}{\lambda^2}$$

$$A_e = P_a A$$

where, P_a - aperture

$$\therefore R_{\max} = \left[\frac{P_t \cdot 4\pi A_e \sigma A_e}{\lambda^2 (4\pi)^2 8 \sin^2 \theta} \right]^{1/4}$$

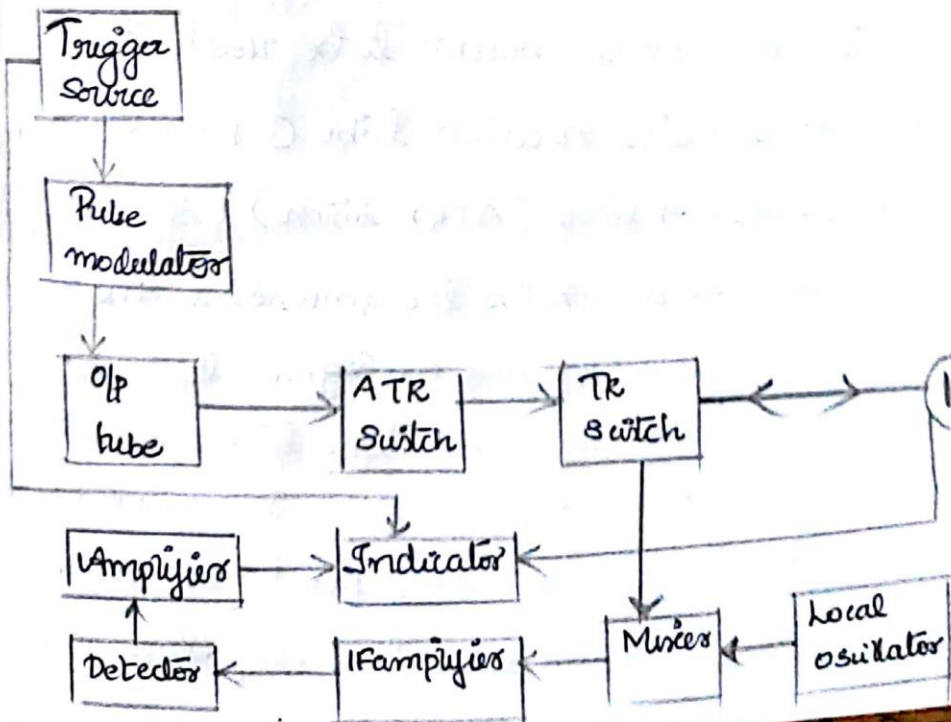
$$R_{\max} = \left[\frac{P_t \sigma A_e^2}{4\pi \lambda^2 8 \sin^2 \theta} \right]^{1/4}$$

Pulsed Radar

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Mon

It is a high power, high freq, radar which transmits pulses towards the target object. The range & resolution of radar depends on pulse repetition frequency. The radio frequency transmitted by the pulsed radar consist of series of equally spaced pulses having duration about 1 μ sec separated by very short intervals.

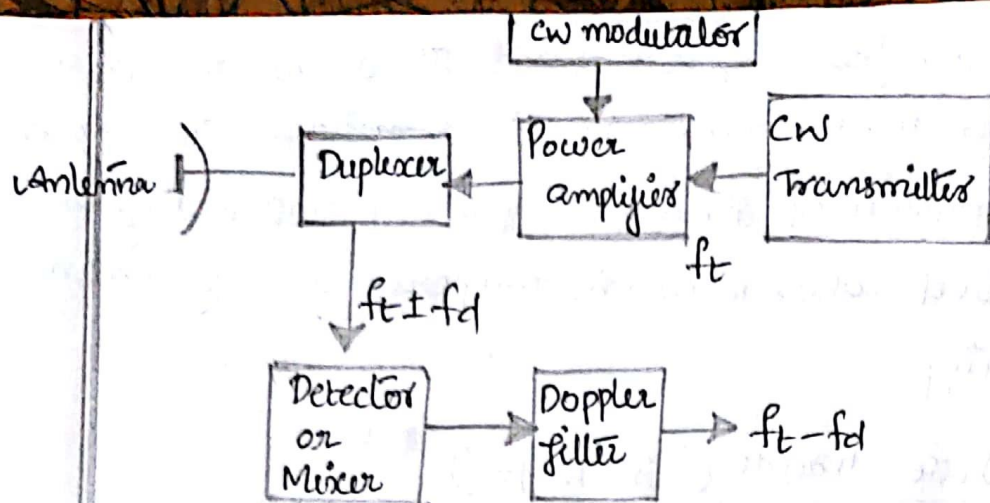


The transmitter may be an oscillator such as a magnetron that is pulsed (turned on & off) by the modulator to generate repetitive train of pulses. The wave form generated by the transmitter travels through the transmission line through the antenna where it is radiated into the space. The trigger source provides pulses for the modulator. The trigger source or synchronizer coordinates the timing for range detection. The pulse modulator provides rectangular voltage pulses which act as the supply voltage to the op tube thus switching it on and off as required. It provides high power to the transmitter to transmit during the transmission period. The op tube may be an oscillator such as a magnetron or amplifier such as klystron amplifier. The pulse modulated wave travels through the duplexer where it is radiated into space. A single parabolic antenna is used for both transmission & reception. The duplexer alternately switches the antenna betⁿ the transmitter & receiver^{sub} so that only one antenna needed to be used. The duplexer consist of transmitter receiver tube (TR switch) & anti-transmitter, receiver tube (ATR switch). The TR switch protects the receiver during the transmission & ATR switch helps in directing the received eco signals to the receiver. The receiver is usually superheterodyne type. The 1st stage of the receiver is a low noise amplifier. The mixer & LO convert the RF up from the amplifier to comparatively lower frequency level called intermediate frequency stage. Thus in a

mixer the carrier frequency is reduced. The detector used is a Schottky barrier diode which extracts the modulated pulse waveform from the IF amplifier op. The detector op is then amplified by an amplifier to a level where it can be properly displayed on an indicator directly.

Continuous Wave Radar (CW Radar)

The CW radar transmits high freq signals continuously. The eco signal is received and processed permanently. It is a type of radar system where a known stable frequency continuous wave radio energy is transmitted and received from reflecting objects. These radars determine the target velocity rather than its location and they are simple, compact and less costly. CW radar works on the principle of Doppler effect. The Doppler effect is the change in frequency that occurs when a source & target are in relative motion. The CW radar uses the Doppler effect to determine the velocity with which the target is moving. The transmitter generates a continuous sinusoidal signal at a frequency ' f_t ' which is radiated by the antenna. On reflection by a moving object, the transmitted signal is shifted by Doppler effect by an amount $\pm f_d$ when, the target is moving towards the transmitter or source f_d will be +ve. If the target is moving away from the source f_d will be -ve. The detector or mixer multiplies the eco signal at a freq $f_t \pm f_d$. The Doppler filter allows the difference freq component to pass through & rejects the higher freq. The op of the detector is amplified by a power amplifier.



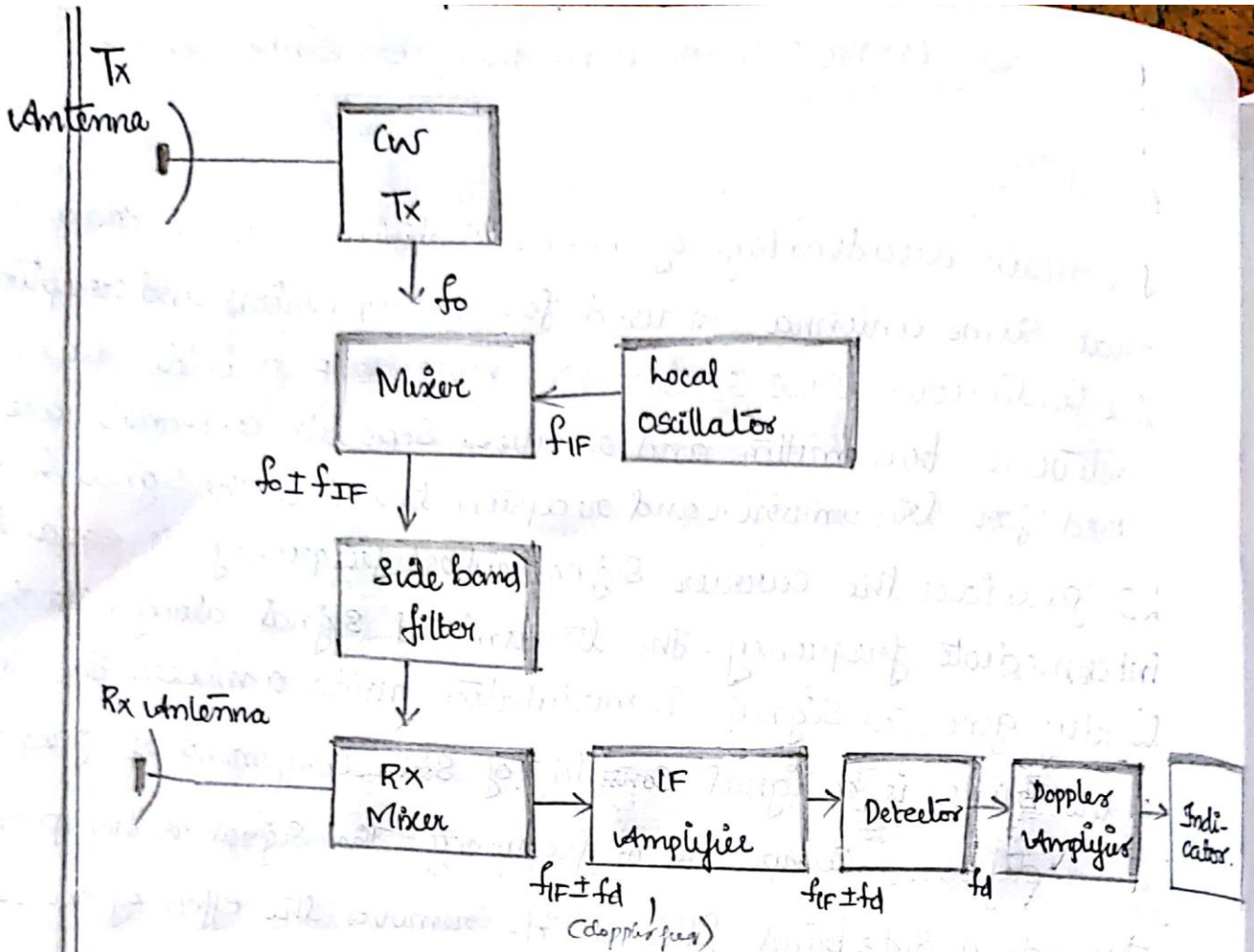
Comparison between Pulsed Radar & CW Radar

<u>Pulsed Radar</u>	<u>CW Radar</u>
1. In this system pulse modulated signal is used for transmission.	1. This system uses continuous wave modulated signals for transmission.
2. Duplexer is used to use common antenna for transmission & reception.	2. Circulator is used to separate antenna for transmission & reception.
3. It indicates the range of the radar.	3. It indicates the velocity of the moving target.
4. Higher transmission power.	4. Lower transmission power.
5. Complicated & complex.	5. Simple.
6. Does not use Doppler shift.	6. Uses Doppler shift for working.

Continuous Wave Radar With Non Zero Intermediate

Frequency

The main disadvantage of normal continuous wave radar is that same antenna is used for transmission and reception. The continuous wave radar with non zero IF provides isolation between transmitter and receiver. Separate antennas are used for transmission and reception to reduce the signal leakage. LO provides the carrier signal whose frequency is equal to intermediate frequency. The transmitted signal along with the locally generated signal is modulated inside a mixer. The op of the mixer is a signal consisting of ~~some~~ ^{sum} components of frequency and difference components of frequency. The signal is then passed through a sideband filter which removes the effect of noise & reduces the receiver sensitivity. The op of the sideband filter is fed onto a receiver mixer where the doppler frequency is added along with the original frequency of the signal. The doppler freq. can be +ve or -ve depending on the location of the target. The op of the receiver mixer is then amplified and passed onto a detector. The detector removes the IF component from the received signal and passes only the doppler freq component. The doppler freq component is then amplified by a doppler amplifier. The op of the doppler amplifier is fed onto an indicator which provides the location of the target depending on the value of doppler frequency.

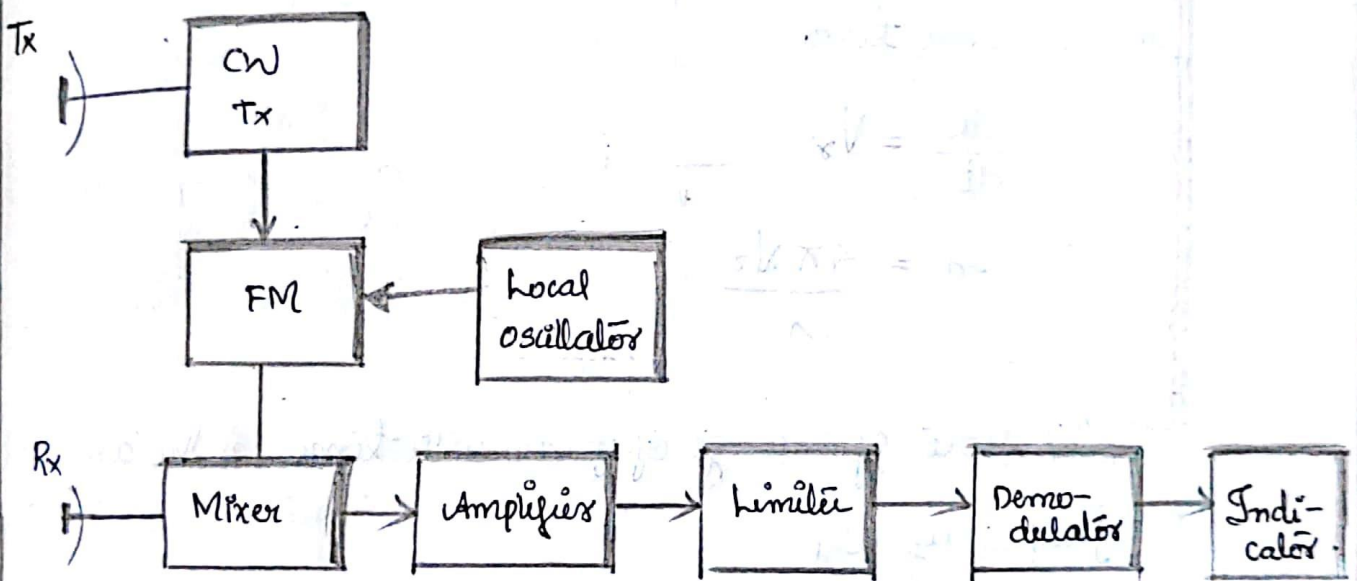


Frequency Modulated Continuous Wave Radar

The frequency modulated Continuous wave radar (FMCW) radar is capable of measuring the relative velocity and range of the target depending on the BW of the signal.

The time taken by the transmitted signal to reach the receiver and return back to the transmitter is calculated. In the transmitting section the transmitted signal is freq. modulated by using a FM modulator. The modulated signal is propagated on to the receiver. In the receiving section the freq modulated signal is mixed with a high BW signal in a mixer. The op of the mixer is a signal having large BW. This signal is amplified and applied to a limiter circuit. The limiter circuit removes the unwanted freq

Components. This signal is demodulated and applied to an indicator which provides the location of the target. The location of the target is decided by comparing the signal value with a predefined threshold value. If the value of the signal is greater than the threshold value it is decided that a target is present at a certain distance. If the value of the signal is less than the threshold value it is decided that noise is present.



11/18
Friday

Equation for Doppler Frequency Shift

The Doppler effect is the process by which the target or object is detected with change in frequencies. The change in frequency can be +ve or -ve depending on the location of the target. If the target is moving towards the radar the freq. shift will be +ve and if the target is moving away from the radar the frequency shift will be -ve. If the range to the target is R and the wavelength is λ , then the total no. of signals in propagation from radar to the target and return is $\frac{2R}{\lambda}$. The total phase change $\phi = 2\pi \times (\text{Total no. of signals})$ i.e., $\phi = 2\pi \times \frac{2R}{\lambda}$

$$\text{ie } \phi = \frac{4\pi R}{\lambda}$$

If the target is in motion the angular frequency is given as $\omega_d = \frac{d\phi}{dt}$

$$\text{ie, } \omega_d = \frac{4\pi}{\lambda} \frac{dR}{dt}$$

The term $\frac{dR}{dt}$ is the radial velocity or rate of change of range with time.

$$\frac{dR}{dt} = V_r$$

$$\therefore \omega_d = \frac{4\pi V_r}{\lambda}$$

→ The rate of change of phase with time is the angular frequency ω_d .

$$\omega_d = 2\pi f_d$$

where, f_d is the doppler frequency shift.

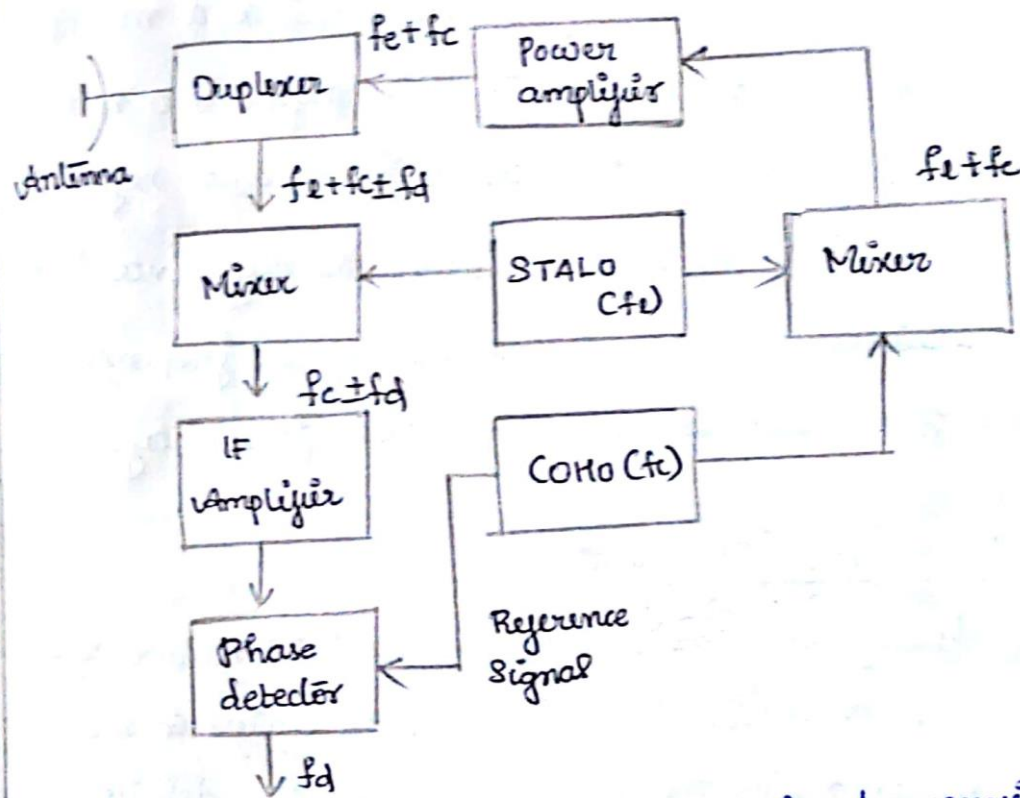
$$f_d = \frac{2V_r}{\lambda}$$

$$\text{ie, } f_d = \frac{2f V_r}{c}$$

Moving Target Indication (MTI) Radar (with power amplifier)

The radars are used to detect targets in the presence of noise. In practical cases the radars have to deal with more receiver noise when detecting targets since there will be ^{echos} a large returning from a

natural environment. These echoes are known as 'clutters'. Clutter echoes can be orders of larger magnitude. MTI radar is a type of radar that employs the doppler shift for detecting moving objects in the presence of clutters. These type of radars can detect the location of object more efficiently compared to normal radars.

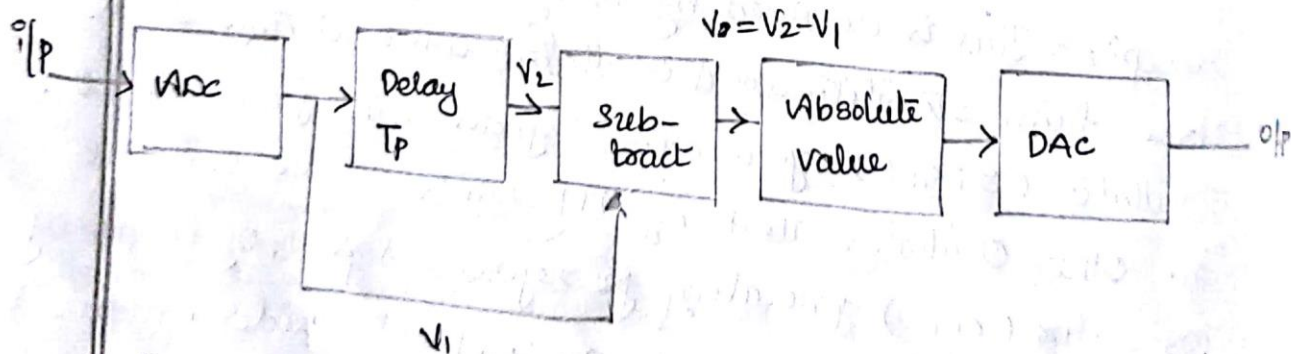


The MTI radar uses a single antenna for transmission and reception. This is achieved by using a duplexer. The radar uses high stability local oscillators called stable local oscillators (STALO) generating signals with frequency f_c . The other oscillator used by MTI radar is the coherent oscillator (COHO) generating reference signals of frequency f_c . The frequencies f_c and f_c are combined inside a mixer producing the sum of the signals. This signal is amplified by a power amplifier and is transmitted through antenna. The combination of STALO and COHO is called the receiver exciter.

portion of the MTI radar. This combination is used to ensure better stability for the operation of radar. When the amplified signal is transmitted the doppler frequency component f_d is added along with the signal. The o/p of the mixer will be an IF signal. The IF signal is amplified by an IF amplifier which acts as a matched filter. The phase detector following the IF amplifier is a mixer like device that combines the received signal and reference signal from the COHO so as to produce the difference between the received signal and reference signal frequencies. The o/p of the phase detector is the doppler frequency.

Delay Line Canceller

The delay line canceller is used in MTI radar system and it works as a filter. It eliminates the clutter in received signal there by improving the resolution of target detection. The delay line canceller subtracts echos from the successive signals.



The o/p of the MTI radar is given as I/P to delay line canceller. The I/P signal is converted to its equivalent digital value by an analog to digital converter. The signal is delayed

which is achieved by storing the radar or delaying the pulse transmission. The original signal and delayed version of the signal are then subtracted to produce an o/p signal which is free from echoes. The absolute value of the difference signal is then taken and is fed to a digital to analog converter to produce the o/p signal.

$$\text{Let } V_1 = k \sin [2\pi f_d t - \phi]$$

$$V_2 = k \sin (2\pi f_d (t - T_p) - \phi)$$

$$\therefore V = V_2 - V_1$$

$$= k \sin (2\pi f_d (t - T_p) - \phi) - k \sin [2\pi f_d t - \phi]$$

$$= 2k \left\{ \sin \left[\frac{2\pi f_d (t - T_p) - \phi - (2\pi f_d t - \phi)}{2} \right] \right.$$

$$\left. \cos \left[\frac{2\pi f_d (t - T_p) - \phi + 2\pi f_d t - \phi}{2} \right] \right\}$$

$$\begin{aligned} \sin A - \sin B &= 2 \sin \frac{A-B}{2} \\ \cos A + \cos B &= 2 \cos \frac{A+B}{2} \end{aligned}$$

$$= 2k \left\{ \sin \left[\frac{2\pi f_d t - 2\pi f_d T_p - \phi + 2\pi f_d t + \phi}{2} \right] \right.$$

$$\left. \cos \left[\frac{2\pi f_d t - 2\pi f_d T_p - \phi + 2\pi f_d t - \phi}{2} \right] \right\}$$

$$= 2k \sin (2\pi f_d t - \pi f_d T_p) \cos \left(\frac{4\pi f_d t - 2\pi f_d T_p - 2\phi}{2} \right)$$

$$= 2k \sin (-\pi f_d T_p) \cos (2\pi f_d t - \pi f_d T_p - \phi)$$

$$= 2k \sin (\pi f_d T_p) \cos (2\pi f_d t - \pi f_d T_p - \phi)$$

Considering only the magnitude of the signal

$$V = 2k \sin (\pi f_d T_p) \cos (2\pi f_d t - \pi f_d T_p - \phi)$$

$$i.e., V = H(f) \times \cos(2\pi f_d t - \pi f_d T_p - \phi)$$

where, $H(f)$ is the amplitude of the transmitted signal and

$$H(f) = 2\pi \sin(\pi f_d T_p)$$

→ The delay line cancellers have a disadvantage called 'blind Speed' where the target will not be detected and there will be an uncanceled clutter remaining interfering with target detection process. The Doppler shift frequency in presence of clutter is defined as

$$f_d = n f_p \quad \text{--- (1)}$$

The general equation for Doppler shift frequency is

$$f_d = \frac{2 f_t V_r}{c} \quad \text{--- (2)}$$

equating eqⁿ (1) & (2)

$$n f_p = \frac{2 f_t V_r}{c}$$

$$\therefore V_r = \frac{c n f_p}{2 f_t}$$

$$= \frac{n f_p \lambda}{2}$$

$$V_r = \frac{n \lambda}{2 T_p}$$

V_r , represents the velocity for calculating blind speed
four methods for reducing the effects of blind speed are

- Operate radar at longer wavelengths.
- Operate radar with high pulse repetition frequency



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→ They are easy to maintain & have long life.

5. Magnetron

→ They are smaller in size and utilizes low voltages for operation.

The klystron, travelling wave tube & magnetron are called slow wave devices in which the phase velocity of electromagnetic wave is slow compared to velocity of electrons. The Gyrotron is a fast wave device in which the phase velocity of the EM wave is higher than velocity of electron.

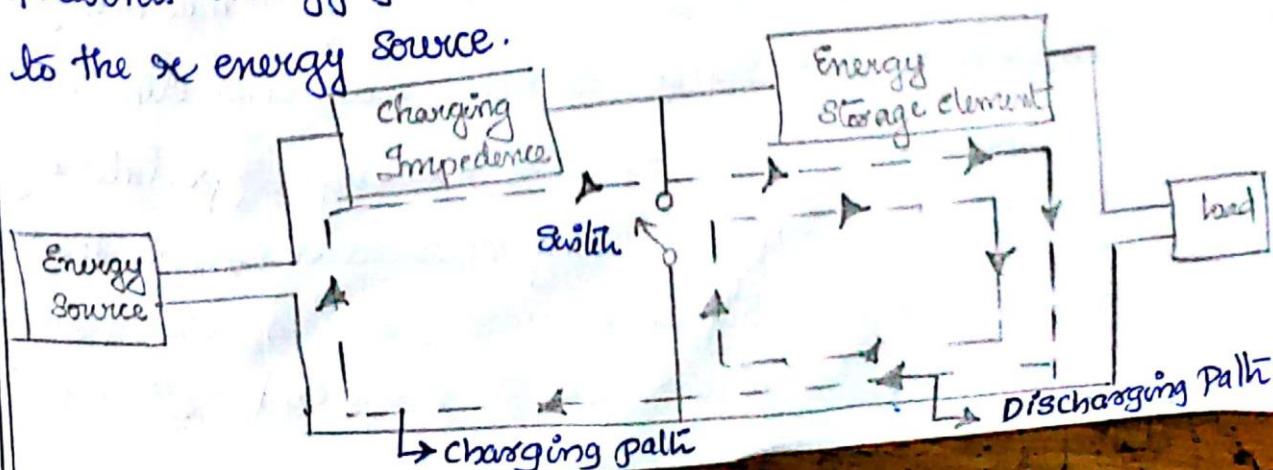
Radar Modulators

→ The function of the modulator is to turn on and off the transmitter to generate the desired waveform. When the waveform is a pulse the modulator is called pulse modulator or pulser.

→ Energy from the external power source is accumulated in the energy storage element.

→ The charging impedance limits the rate at which energy is delivered to storage element. When the pulse is ready to be formed the switch is closed and the stored energy is discharged through the load.

→ During the discharge path of the cycle the charging impedance prevents energy from the storage element being returned to the energy source.



Line Type Modulator

In this device a delay line ^{collection of small capacitors} or pulse forming π network is used as storage element. The switch can be a Silicon Controlled Rectifier (SCR) that can initiate the discharge of pulse forming π network to form a rectangular pulse. The shape & duration of the pulse are determined by passive elements of pulse forming π network. This type of modulator is simple, compact in size and can tolerate abnormal load conditions.

Active Switch Modulator

The switch in the active switch modulator controls both the beginning and end of the pulse. The energy storage element is a capacitor. Large capacitance can be obtained with a collection of capacitors known as capacitor bank.

The active switch modulator permits more flexibility than line type modulators.

Radar Receivers

The function of the receiver is to extract the weak echoes signals and amplify them. It employs a matched filter to maximize the signal to noise ratio and eliminate unwanted signals. The radar receiver is always Superhetrodyne in nature. The Superhetrodyne receiver converts the RF input signal to an IF signal to achieve desired BW, gain and stability. The first stage or front end of a Superhetro-

dyne receiver is a low noise amplifier.

Receiver Noise Figure

It is defined as the measure of noise produced by a practical receiver compared to noise of an ideal receiver. The noise figure is expressed as

$$F_n = \frac{N_{out}}{k T_0 B_n G}$$

where, N_{out} - available op noise power.

k - Boltzmann's Constant

G - Gain.

T_0 - standard temperature

B_n - noise BW.

If additional noise is introduced by practical n/w then,

$$\text{noise figure} = F_n = \frac{k T_0 B_n G + \Delta N}{k T_0 B_n G}$$

$$F_n = 1 + \frac{\Delta N}{k T_0 B_n G}$$

Noise Figure in Cascade Networks

Consider two n/w's in cascade each with same noise and BW but with different gain. Let the noise from the 1st n/w be $F_1 k T_1 B_n G_1 G_2$ and the noise from the 2nd n/w be $(F_2 - 1) k T_0 B_n G_2$.

∴ the op noise, $N_{out} = \text{noise from 1st n/w} + \text{noise from 2nd n/w}$

$$\therefore N_{out} = F_1 k T_1 B_n G_1 G_2 + (F_2 - 1) k T_0 B_n G_2$$

$$\text{Let, } N_{out} = F_0 k T_0 B_n G_1 G_2$$

∴ the above eqn becomes

$$F_0 k T_0 B_n G_1 G_2 = F_1 k T_1 B_n G_1 G_2 + (F_2 - 1) k T_0 B_n G_2$$

$$F_0 = (F_1 k T_1 B_n G_1 G_2 + (F_2 - 1) k T_0 B_n G_2) / k T_0 B_n G_1 G_2$$

provides two o/p frequencies that are sum and difference of two i/p frequencies.

$$f_c, f_{RF} \pm f_{LO}$$

The difference frequency $f_{RF} - f_{LO}$ is the desired IF frequency.

The sum frequency $f_{RF} + f_{LO}$ is removed by filtering. If the RF signal frequency is greater than local oscillator frequency the o/p IF frequency will be

$$f_{IF} = f_{RF} - f_{LO}$$

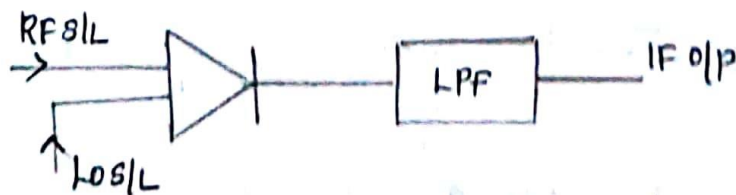
This is known as desired frequency.

→ If the RF signal frequency is less than local oscillator freq, then the o/p IF signal will be

$$f_{IF} = f_{LO} - f_{RF}. \text{ This is called } \underline{\text{image frequency}}.$$

a) Single Ended Mixer

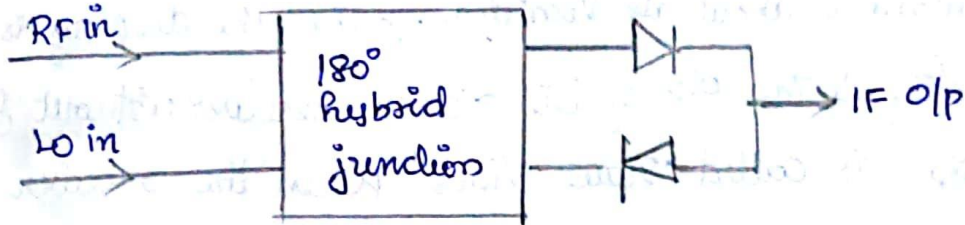
It uses a single diode. The LO signal is inserted through a directional coupler. A low pass filter following the diode allows the IF signal to pass through it by rejecting high frequencies. The diode of the mixer can produce nonlinearities called spurious responses which are unwanted intermediate frequency components.



b) Balanced Mixer

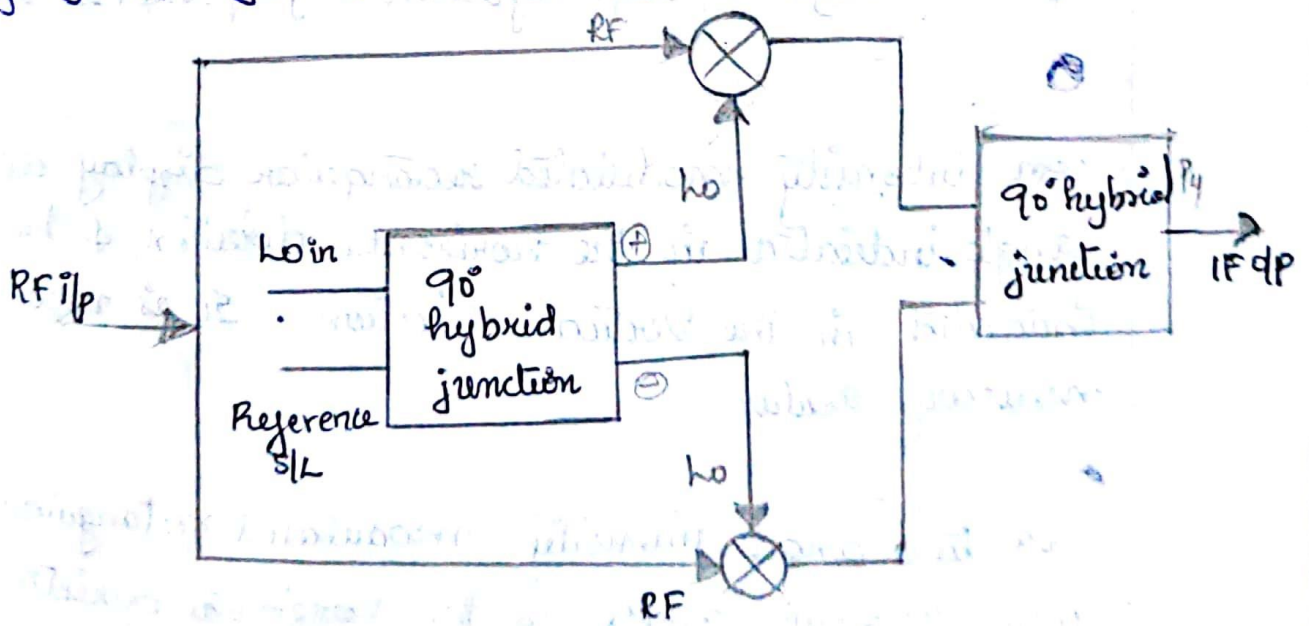
The balanced mixer can be considered as two single ended mixers in parallel & 180° out of phase. The LO signal is

Applied to one port and RF signal is applied to second port. The signals inserted at these two ports appear in the 3rd port as Sum and 4th port as difference. The IF signal is obtained by subtracting the o/p of the two diode mixers. A double balanced mixer utilises four diodes connected in ring or bridge to produce the IF signal output.



c) Image Rejection Mixer

In an image rejection mixer the RF signal is split and fed to two mixers. The LO signal is fed into one port of a 90° hybrid junction that produces 90° phase difference between the two outputs to the two mixers. The IF hybrid junction provides another 90° phase difference in such a manner that signal frequency & image frequency are separated.



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Friday

Radar Displays

The radar display has the important purpose of visually presenting the o/p of radar receiver in a form such that an operator could accurately detect the presence of a target and extract information about its location. When the display is connected directly to the o/p of the radar receiver without further processing the o/p is called 'raw video'. When the receiver o/p is processed by a detector it is called Synthetic video or processed video.

Types of Displays.

● A Scope

A deflection modulated rectangular display in which vertical deflection is proportional to amplitude of receiver o/p and horizontal deflection is proportional to range.

● D Scope

An intensity modulated rectangular display with azimuth angle indicated in the horizontal direction & the range indicated in the vertical direction. It is used in military radar.

● C Scope

A two angle intensity modulated rectangular display with azimuth angle in the horizontal direction & the elevation angle in the vertical direction.

● E Scope Display

An intensity modulated rectangular display with

range indicated is the horizontal coordinate & elevation angle in the vertical coordinate.

o Plan Position Indicator (PPI)

An intensity modulated circular display in which echo signals from the reflecting objects are shown with range and azimuth angle displayed in polar coordinates.

o Range Height Indicator (RHI)

An intensity modulated rectangular display with height of the target or altitude of the target along vertical axis and range along horizontal axis.

The cathode ray tube displays were used in early stages of the radar, the deflection modulated CRT, such as A scope display in which a target is indicated by deflection of the electron beam was commonly used. The other type of CRT called intensity modulated CRT in which an echo is indicated by intensity of the electron beam & presenting a display on the CRT was also in use.

Tutorial Problems:

Module 1: Problems

- 1) A reflex klystron is operating at 100 GHz . If the mode operating in the tube corresponds to integer $n=4$, determine the transit time of the electron in the repeller space.

Soln: The transit time (T) is $\approx n + 3/4$. Given $n=4$

$$\therefore T = 4 + 3/4 = 19/4 \text{ cycles.}$$

for $f = 100 \text{ GHz}$, one cycle corresponds to $1/10^{10} = 10 \text{ ps}$. [ie $T = 1/f = \frac{1}{10 \times 10^9}$]

$$\therefore T = 19/4 \times 10 = 47.5 \text{ ps.}$$

- 2) A two cavity klystron amplifier has the following parameters [Similar to Dec. 2010]

$$V_0 = 1000 \text{ V}; R_0 = 20 \text{ k}\Omega; I_0 = 20 \text{ mA}; f = 3 \text{ GHz.}$$

Gap spacing in either cavity: $d = 1 \text{ mm}$.

Spacing between the two cavities: $L = 4 \text{ cm}$.

Effective shunt impedance, excluding beam loading $R_{sh} = 30 \text{ k}\Omega$.

- (a) Find the input gap voltage to give maximum voltage V_2 .
 (b) Find the voltage gain, neglecting the beam loading in the output cavity.
 (c) Find the efficiency of the amplifier neglecting beam loading.
 (d) Calculate the beam loading conductance.

Solution: (a) For maximum V_2 , $J_1(x)$ must be maximum. This means $J_1(x) = 0.582$ at $x = 1.841$. The electron velocity just leaving the cathode is $v_0 = (0.593 \times 10^6) \sqrt{V_0} = (0.593 \times 10^6) \sqrt{10}$
 $= 1.88 \times 10^7 \text{ m/s.} \quad - (1)$

→ The gap transit angle is $\theta_g = \frac{\omega d}{v_0} \quad - (2)$

$$\text{Sub. (1) in (2) we get} \quad = \frac{2\pi f \cdot d}{v_0} = 2\pi (3 \times 10^9) \frac{1 \times 10^{-3}}{1.88 \times 10^7} \\ = 1 \text{ rad.}$$

Beam coupling coefficient is

$$\beta_c = \beta_0 = \frac{\sin(\theta_g/2)}{\theta_g/2} = \frac{\sin(1/2)}{1/2} = 0.952.$$

The dc transit angle between the cavities is $\theta_0 = \omega T_0 = \omega \frac{L}{v_0}$

$$\therefore = 2\pi f \cdot \frac{L}{v_0} = 2\pi (3 \times 10^9) \frac{4 \times 10^{-2}}{1.88 \times 10^8} = 40 \text{ rad.}$$

The maximum input voltage V_1 is $V_{1 \max} = \frac{2V_0 X}{\beta_1 \theta_0}$

$$= \frac{2 \times 10^3 \times 1.841}{(0.952)(40)} = 96.7 \text{ V.}$$

(b) The Voltage gain (AV): $= \frac{\beta_0^2 \theta_1}{R_0} \cdot \frac{J_1(x)}{x} \cdot R_{sh}$

$$= \frac{(0.952)^2 (40) \cdot (0.582) (30 \times 10^3)}{2 \times 10^4 \times 1.841} = 17.19.$$

(c) The efficiency.

$$I_2 = 2I_0 J_1(x) = 2 \times 20 \times 10^{-3} \times 0.583 = 23.32 \times 10^{-3} \text{ A.}$$

$$V_2 = \beta_0 I_2 R_{sh}$$

$$= (0.952) (23.32 \times 10^{-3}) (30 \times 10^3) = 666 \text{ V.}$$

$$\text{Efficiency} = \frac{\beta_0 I_2 V_2}{2I_0 V_0} = \frac{(0.952)(23.32 \times 10^{-3})(666)}{2(20 \times 10^{-3})(10^3)} = 36.9\%$$

(d) The Beam loading conductance G_B is.

$$G_B = \frac{G_0}{2} \left(\beta_0^2 - \beta_0 \cos \frac{\theta_0}{2} \right), \text{ where } G_0 \text{ is beam conductance.}$$

$$\therefore G_0 = \frac{I_0}{V_0} = \frac{20 \times 10^{-3}}{1000} = 20 \times 10^{-6}$$

$$\text{hence } G_B = \frac{20 \times 10^{-6}}{2} \left[(0.952)^2 - (0.952) \cos(28.6^\circ) \right]$$

$$= 8.8 \times 10^{-7} \text{ mho.}$$

$$\left[\because \text{W.K.T. } 1 \text{ rad} = 57.3^\circ \right]$$

$$\& \theta_0/2 = \frac{40}{2} = 20^\circ$$

$$= 28.6^\circ$$

Prob 3: A reflex klystron operates under the following conditions
 $V_0 = 600 \text{ V}$; $L = 1 \text{ mm}$, $R_{sh} = 15 \text{ k}\Omega$, $e/m = 1.759 \times 10^{11}$
 $f_r = 9 \text{ GHz}$.

The tube is oscillating at f_r , at the peak of the $n=2$ mode or $1 \frac{3}{4}$ mode. Assume that the transit time through the gap and beam loading can be neglected.

- Find the value of the repeller voltage V_r .
- Find the direct current necessary to give a microwave gap voltage of 200 V .
- What is the electronic efficiency under this condition.

Solution:

$$(a) \quad \frac{V_0}{(V_r + V_0)^2} = \frac{e}{m} \frac{2\pi n - (\pi/2)^2}{8\omega^2 L^2}$$

$$= (1.759 \times 10^{11}) \frac{2\pi(2) - (\pi/2)^2}{8(2\pi \times 9 \times 10^9)^2 (10^{-3})^2}$$

$$= 0.832 \times 10^{-3}$$

$$(V_r + V_0)^2 = \frac{600}{0.832 \times 10^{-3}} \Rightarrow V_r + V_0 = \sqrt{0.721 \times 10^6}$$

$$= 0.721 \times 10^3$$

$$V_r = (0.849 \times 1000) - 600$$

$$\boxed{V_r = 249 \text{ V}}$$

(b) Assume that $P_0 = 1$,

$$V_2 = I_2 R_{sh}$$

$$= 2 I_0 J_1(x_1') R_{sh}; \quad \text{Direct current } 'P_0'$$

$$I_0 = \frac{V_2}{2 J_1(x_1') R_{sh}}$$

$$\Rightarrow \frac{200}{2(0.582)(15 \times 10^3)} = \underline{\underline{11.45 \text{ mA}}}$$

$$(c) \text{ Electronic efficiency} = \frac{2X'_1 J_1(X'_1)}{2\pi n - \pi/2} = \frac{2(1.841)(0.582)}{2\pi(2) - \pi/2} = 19.49\%$$

$$\text{for } n = 1\frac{3}{4} \quad (or) = 22.7\%$$

Prob 4: A reflex klystron is operated at 96 Hz with dc beam voltage 600V for $1\frac{3}{4}$ mode, repeller space length 1mm, dc beam current 10mA. The beam coupling co-efficient is assumed to be 1. Calculate (i) the repeller voltage (ii) output power and (iii) electronic efficiency.

Solution: Given: $f_0 = 96 \text{ Hz}$, $N = 1\frac{3}{4}$, $V_0 = 600 \text{ Volt}$, $L = 1 \text{ mm}$, $I_0 = 10 \text{ mA}$, $V_R = ?$, $P_{RF} = ?$, $\eta = ?$.

(i) w.k.t: $|V_R| = 6.74375 \times 10^{-6} \times f \times \frac{L}{N} \sqrt{V_0} - V_0$ (f in Hertz, L in meter).

$$= 6.74 \times 10^{-6} \times 1 \times 10^{-3} \times 9 \times 10^9 \times \frac{\sqrt{600}}{1\frac{3}{4}} - 600$$

$$= 249 \text{ V.}$$

(ii) $P_{RF \text{ max}} = \frac{0.398 V_0 I_0}{N} = \frac{0.398 \times 600 \times 10 \times 10^{-3}}{1\frac{3}{4}} \text{ Watts.}$

$$= 0.2274 \times 600 \times 10 \times 10^{-3}$$

$$= 1.3644 \text{ Watts.}$$

(iii) $\eta_{\text{max}} = \frac{X J_1(X)}{\pi N} = \frac{0.398}{N} = \frac{0.398}{1\frac{3}{4}} = 22.74\%$

Q. Find the f_c of the 1st 5 lowest modes of an air filled rectangular cavity of dimensions $5\text{cm} \times 4\text{cm} \times 2.5\text{cm}$ ($a \times b \times d$)

Ans: $TE_{101} \Rightarrow$ ✓

TE_{101}

$$f_c = \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2}$$

$$= \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{1}{5}\right)^2 + 0 + \left(\frac{1}{2.5}\right)^2} \Rightarrow \frac{1}{2\sqrt{\mu\epsilon}} \times \sqrt{\left(\frac{1}{5 \times 10^{-2}}\right)^2 + \left(\frac{1}{2.5 \times 10^{-2}}\right)^2}$$

$$= 149.89 \times 10^6 \times 44.721$$

$$= \underline{\underline{6.716 \text{ GHz}}}$$

Q. $TM_{110} =$

$$TE_{110} \quad 149.89 \times 10^6 \times \sqrt{\left(\frac{1}{0.05}\right)^2 + \left(\frac{1}{0.04}\right)^2}$$

$$= 4.796 \times 10^9 \text{ Hz}$$

$$= \underline{\underline{4.796 \text{ GHz}}}$$

$$TM_{111} = 149.89 \times 10^6 \times \sqrt{\frac{1}{0.05^2} + \frac{1}{0.04^2} + \frac{1}{0.025^2}}$$

TE_{111}

$$= \underline{\underline{7.69 \text{ GHz}}}$$

$$\begin{aligned}
 TE_{011} &= 149.89 \times 10^6 \times \sqrt{\frac{1}{0.04^2} + \frac{1}{0.025^2}} \\
 TM_{011} &= 7.08 \text{ GHz} \\
 TE_{111} &= 149.89 \times 10^6 \times \sqrt{\frac{1}{0.03^2} + \frac{1}{0.04^2} + \frac{1}{0.025^2}} \\
 &= 7.69 \text{ GHz}
 \end{aligned}$$

TM_{111} and TE_{111} are degenerate mode

$\therefore TM_{110}$ is the dominant mode.

TWT Problem: A helix travelling-wave tube operates at 4 GHz under a beam voltage 10KV and beam current 500mA. If the helix impedance is 25ohm and the interaction length is 20cm. find the output power gain in dB.

Solution! Given $V_0 = 10\text{KV}$, $I_0 = 500\text{mA}$, $Z_0 = 25\text{ohm}$,
 $f = 4\text{GHz}$, $l = 20\text{cm}$.

$$\begin{aligned}
 \therefore v_0 &= 0.593 \times 10^6 \sqrt{V_0} \\
 &= 0.593 \times 10^6 (10 \times 10^3)^{1/2} \\
 &= 0.593 \times 10^8 \text{ m/sec}
 \end{aligned}$$

$$N = l / \lambda_e \quad [N \text{ is the length of the interaction region in wave length}]$$

[l is the length of the slow-wave structure in meters and .

$$\lambda_e = \frac{2\pi v_0}{\omega} \quad \text{where } v_0 = \sqrt{\frac{R_{eff}}{L}} v_0]$$

$$\text{hence } N = \frac{l \cdot \omega}{2\pi v_0}$$

$$= \frac{0.2 \times 2\pi \times 4 \times 10^9}{2\pi \times 0.593 \times 10^8}$$

$$= 13.49.$$

$$C = (I_0 Z_0 / 4 V_0)^{1/3}$$

$$= \left[\frac{500 \times 10^{-3} \times 25}{4 \times 10 \times 10^3} \right]^{1/3}$$

$$= 0.068$$

Therefore $A_p = -9.54 + 47.3 \times 13.49 \times 0.068$

$$= 33.85 \text{ dB}$$

Power gain $A_p = 33.85 \text{ dB}$

Example 10.6 A Si microwave transistor has reactance 1 ohm, transit time cut-off frequency 4 GHz, maximum E-field 1.6×10^5 V/m and saturation drift velocity 4×10^5 cm/s. Determine maximum allowable power.

Solution

Given

$$X_c = 1 \text{ ohm}, E_m = 1.6 \times 10^5 \text{ V/m}$$

$$f_T = 4 \text{ GHz}, v_s = 4 \times 10^5 \text{ cm/s}$$

$$P_m X_c = \left(\frac{E_m v_s}{2\pi f_T} \right)^2$$

or,

$$P_m = \left(\frac{E_m v_s}{2\pi f_T} \right)^2 \frac{1}{X_c}$$

$$= \left(\frac{1.6 \times 10^5 \times 4 \times 10^5}{2 \times 3.14 \times 4 \times 10^9} \right)^2$$

$$= 6.48 \text{ watts}$$

Biasing circuits Microwave transistor biasing

Example 11.1 Calculate the maximum range of a radar system which operates at 3 cm with a peak pulse power of 600 kW if its antenna is 5 m^2 , minimum detectable signal is 10^{-13} W and the radar cross sectional area of the target is 20 m^2 .

Solution. $\lambda = 3 \text{ cm}$; $P_r = 600 \text{ kW}$, $S_{\min} = 10^{-13} \text{ W}$, $A_e = 5 \text{ m}^2$, $\sigma = 20 \text{ m}^2$, $R_{\max} = ?$

$$R_{\max} = \left[\frac{P_r \cdot A_e^2 \cdot \sigma}{4\pi \lambda^2 \cdot S_{\min}} \right]^{1/4} = \left[\frac{600 \times 10^3 \times 5^2 \times 20}{4 \times \pi \times (3 \times 10^{-2})^2 \times 10^{-13}} \right]^{1/4} = 717.657 \text{ km}$$

In nautical miles; $1 \text{ nm} = 1.853 \text{ km}$,

$$R_{\max} = \frac{717.657}{1.853} = 387 \text{ nm}$$

Example 11.2 A 10 GHz radar has the following characteristics, peak transmitted power = 250 kW; power gain of antenna = 2500; minimum detectable peak signal power by receiver = 10^{-14} watts; cross sectional area of the radar antenna = 10 m^2 .

If this radar were to be used to detect a target of 2 m^2 equivalent cross section, find the maximum range possible.

Solution. Given

$P_r = 250 \text{ kW}$; $G = 2500$; $S_{\min} = 10^{-14} \text{ W}$; $A_e = 10 \text{ m}^2$; $\sigma = 2 \text{ m}^2$; $f = 10 \text{ GHz}$

$$\lambda = c/f = \frac{3 \times 10^8}{10 \times 10^9} = 0.03 \text{ m}$$

$$R_{\max} = \left[\frac{P_r \cdot G \cdot A_e \cdot \sigma}{(4\pi)^2 S_{\min}} \right]^{1/4} = \left[\frac{250 \times 10^3 \times 2500 \times 10 \times 2}{(4\pi)^2 \times 10^{-14}} \right]^{1/4}$$

Content beyond Syllabus

5G - Introduction

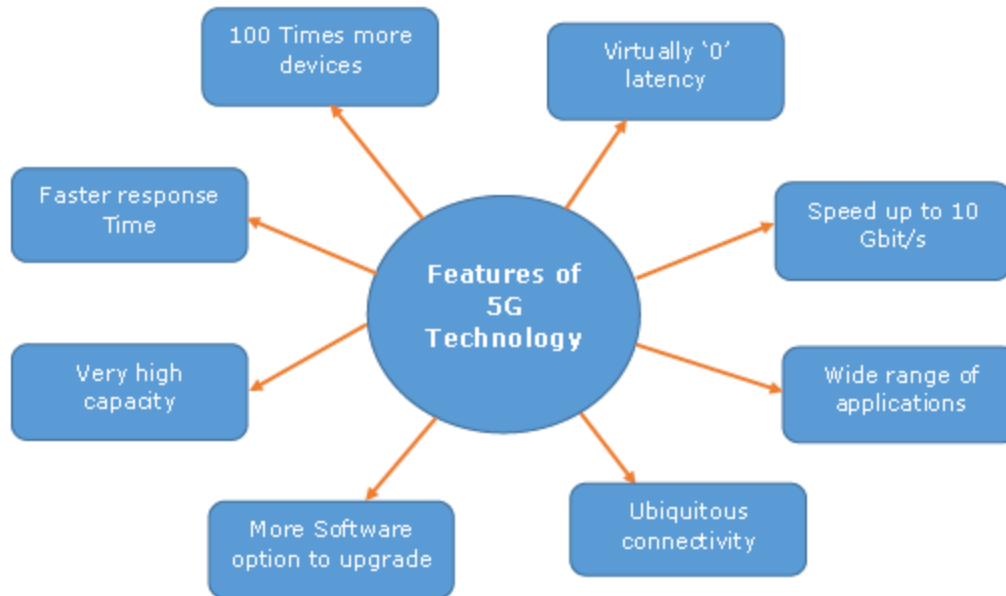
Radio technologies have evidenced a rapid and multidirectional evolution with the launch of the analogue cellular systems in 1980s. Thereafter, digital wireless communication systems are consistently on a mission to fulfil the growing need of human beings (1G, ...4G, or now 5G).



So, this article describes the 5G technology emphasizing on its salient features, technological design (architecture), advantages, shortcomings, challenges, and future scope.

Salient Features of 5G

5th Generation Mobile Network or simply 5G is the forthcoming revolution of mobile technology. The features and its usability are much beyond the expectation of a normal human being. With its ultra-high speed, it is potential enough to change the meaning of a cell phone usability.



With a huge array of innovative features, now your smart phone would be more parallel to the laptop. You can use broadband internet connection; other significant features that fascinate people are more gaming options, wider multimedia options, connectivity everywhere, zero latency, faster response time, and high quality sound and HD video can be transferred on other cell phone without compromising with the quality of audio and video.

5G - Technology

If we look back, we will find that every next decade, one generation is advancing in the field of mobile technology. Starting from the First Generation (1G) in 1980s, Second Generation (2G) in 1990s, Third Generation (3G) in 2000s, **Fourth Generation (4G)** in 2010s, and now Fifth Generation (5G), we are advancing towards more and more sophisticated and smarter technology.



What is 5G Technology?

The 5G technology is expected to provide a new (much wider than the previous one) frequency bands along with the wider spectral bandwidth per frequency channel. As of now, the predecessors (generations) mobile technologies have evidenced substantial increase in peak bitrate. Then — how is 5G different from the previous one (especially 4G)? The answer is — it is not only the increase in bitrate made 5G distinct from the 4G, but rather 5G is also advanced in terms of –

- High increased peak bit rate
- Larger data volume per unit area (i.e. high system spectral efficiency)
- High capacity to allow more devices connectivity concurrently and instantaneously
- Lower battery consumption
- Better connectivity irrespective of the geographic region, in which you are
- Larger number of supporting devices
- Lower cost of infrastructural development
- Higher reliability of the communications

As researchers say, with the wide range of bandwidth radio channels, it is able to support the speed up to 10 Gbps, the 5G *WiFi* technology will offer contiguous and consistent coverage – “wider area mobility in true sense.”

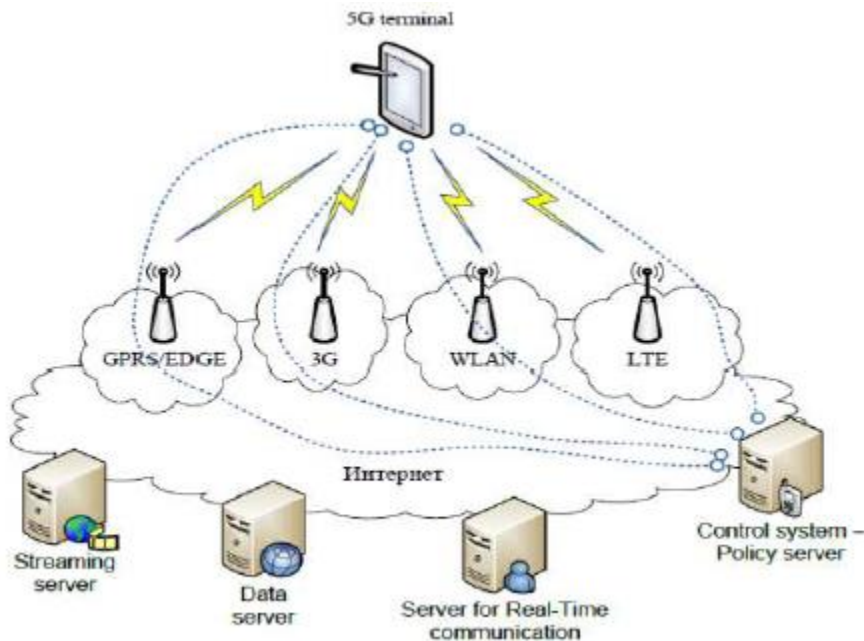
5G - Architecture

Architecture of 5G is highly advanced, its network elements and various terminals are characteristically upgraded to afford a new situation. Likewise, service providers can implement the advance technology to adopt the value-added services easily.

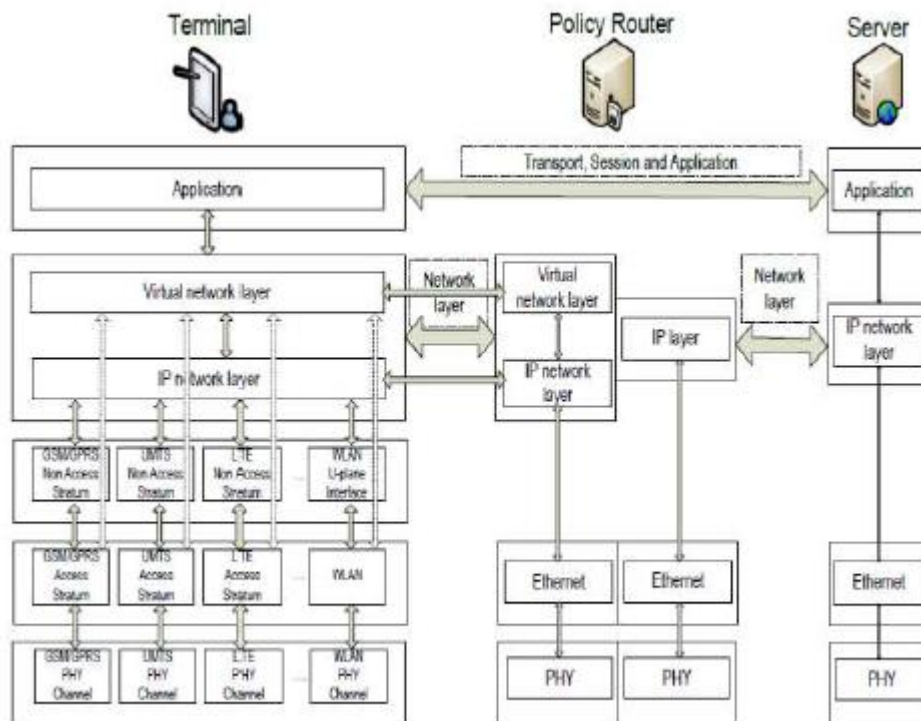
However, upgradeability is based upon cognitive radio technology that includes various significant features such as ability of devices to identify their geographical location as well as weather, temperature, etc. Cognitive radio technology acts as a transceiver (beam) that perceptively can catch and respond radio signals in its operating environment. Further, it promptly distinguishes the changes in its environment and hence respond accordingly to provide uninterrupted quality service.

Architecture of 5G

As shown in the following image, the system model of 5G is entirely **IP** based model designed for the wireless and mobile networks.

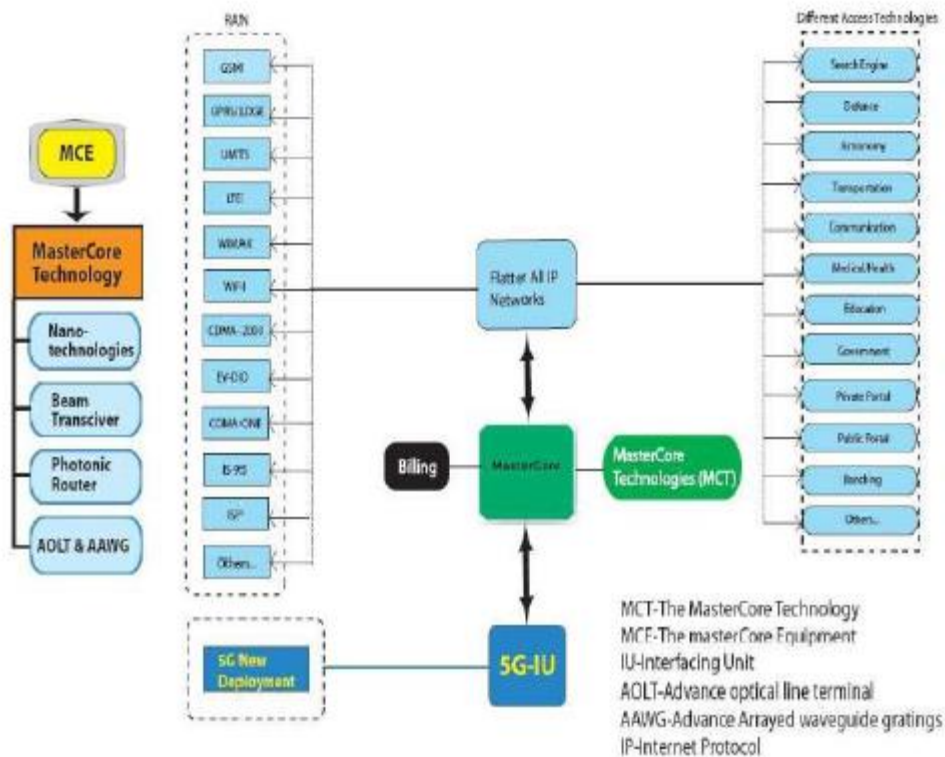


The system comprising of a main user terminal and then a number of independent and autonomous radio access technologies. Each of the radio technologies is considered as the IP link for the outside internet world. The IP technology is designed exclusively to ensure sufficient control data for appropriate routing of IP packets related to a certain application connections i.e. sessions between client applications and servers somewhere on the Internet. Moreover, to make accessible routing of packets should be fixed in accordance with the given policies of the user (as shown in the image given below).



The Master Core Technology

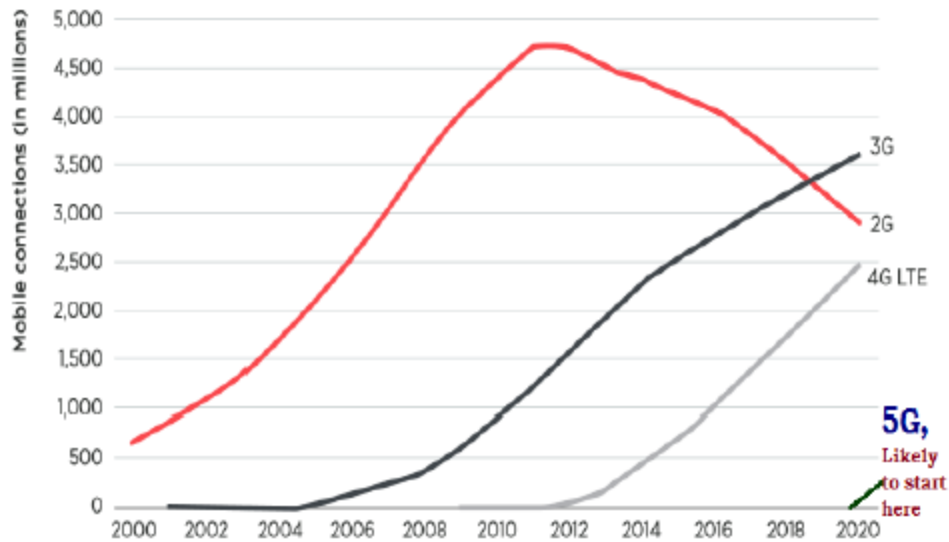
As shown in the Figure 5, the 5G MasterCore is convergence point for the other technologies, which have their own impact on existing wireless network. Interestingly, its design facilitates MasterCore to get operated into parallel multimode including all IP network mode and 5G network mode. In this mode (as shown in the image given below), it controls all network technologies of RAN and Different Access Networks (DAT). Since, the technology is compatible and manages all the new deployments (based on 5G), it is more efficient, less complicated, and more powerful.



Surprisingly, any service mode can be opened under 5G New Deployment Mode as World Combination Service Mode (WCSM). WCSM is a wonderful feature of this technology; for example, if a professor writes on the white board in a country – it can be displayed on another white board in any other part of the world besides conversation and video. Further, a new services can be easily added through parallel multimode service.

5G - Time Period Required

Normally, it is expected that the time period required for the 5G technology development and its implementation is about five years more from now (by 2020). But to becoming usable for the common people in developing countries, it could be even more.



Graph 1 – Showing the Timeline of all previous generation technologies.

Expected Time Length

By considering the multiple utility and various fashionable salient features, researchers are anticipating that this technology will be in use until 2040s.

5G - Applications

5G technology is adorned with many as well as distinct features, which applicability is useful for a wide range people irrespective of their purposes (as shown in the *mweb* image).



Applications of 5G

Some of the significant applications are –

- It will make unified global standard for all.
- Network availability will be everywhere and will facilitate people to use their computer and such kind of mobile devices anywhere anytime.
- Because of the IPv6 technology, visiting care of mobile IP address will be assigned as per the connected network and geographical position.
- Its application will make world real Wi Fi zone.
- Its cognitive radio technology will facilitate different version of radio technologies to share the same spectrum efficiently.
- Its application will facilitate people to avail radio signal at higher altitude as well.

5G - Advancement

Application of 5G is very much equivalent to accomplishment of dream. It is integrated with beyond the limit advance features in comparison to the previous technologies.



Advanced Features

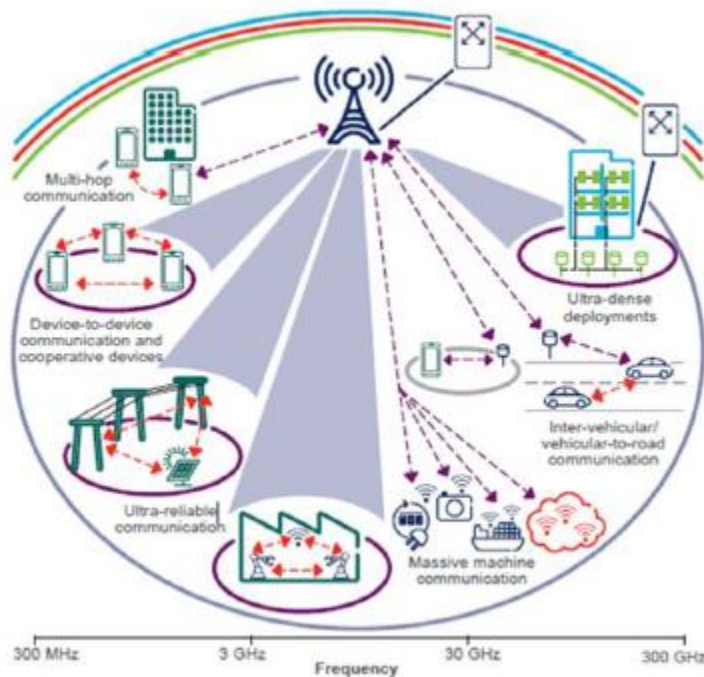
In comparison to previous radio technologies, 5G has following advancement –

- Practically possible to avail the super speed i.e. 1 to 10 Gbps.
- Latency will be 1 millisecond (end-to-end round trip).
- 1,000x bandwidth per unit area.
- Feasibility to connect 10 to 100 number of devices.

- Worldwide coverage.
- About 90% reduction in network energy usage.
- Battery life will be much longer.
- Whole world will be in *wi fi* zone.

5G - Advantages & Disadvantages

5th generation technology offers a wide range of features, which are beneficial for all group of people including, students, professionals (doctors, engineers, teachers, governing bodies, administrative bodies, etc.) and even for a common man.



Important Advantages

There are several advantages of 5G technology, some of the advantages have been shown in the above *Ericsson* image, and many others are described below –

- High resolution and bi-directional large bandwidth shaping.
- Technology to gather all networks on one platform.
- More effective and efficient.
- Technology to facilitate subscriber supervision tools for the quick action.
- Most likely, will provide a huge broadcasting data (in Gigabit), which will support more than 60,000 connections.
- Easily manageable with the previous generations.

- Technological sound to support heterogeneous services (including private network).
- Possible to provide uniform, uninterrupted, and consistent connectivity across the world.

Some Other Advantages for the Common People

Parallel multiple services, such as you can know weather and location while talking with other person.

You can control your PCs by handsets.

Education will become easier – A student sitting in any part of world can attend the class.

Medical Treatment will become easier & frugal – A doctor can treat the patient located in remote part of the world.

Monitoring will be easier – A governmental organization and investigating offers can monitor any part of the world. Possible to reduce the crime rate.

Visualizing universe, galaxies, and planets will be possible.

Possible to locate and search the missing person.

Possible, natural disaster including tsunami, earthquake etc. can be detected faster.

Disadvantages of 5G Technology

Though, 5G technology is researched and conceptualized to solve all radio signal problems and hardship of mobile world, but because of some security reason and lack of technological advancement in most of the geographic regions, it has following shortcomings –

- Technology is still under process and research on its viability is going on.

- The speed, this technology is claiming seems difficult to achieve (in future, it might be) because of the incompetent technological support in most parts of the world.



- Many of the old devices would not be competent to 5G, hence, all of them need to be replaced with new one — expensive deal.
- Developing infrastructure needs high cost.
- Security and privacy issue yet to be solved.

5G - Challenges

Challenges are the inherent part of the new development; so, like all technologies, 5G has also big challenges to deal with. As we see past i.e. development of radio technology, we find very fast growth. Starting from 1G to 5G, the journey is merely of about 40 years old (Considering 1G in 1980s and 5G in 2020s). However, in this journey, the common challenges that we observed are lack of infrastructure, research methodology, and cost.



Still, there are dozens of countries using 2G and 3G technologies and don't know even about 4G, in such a condition, the most significant questions in everyone's mind are –

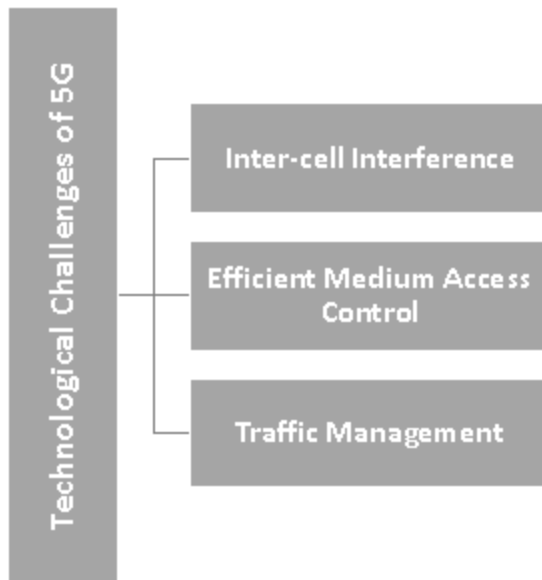
- **How far will 5G be viable?**
- **Will it be the technology of some of the developed countries or developing countries will also get benefit of this?**

To understand these questions, the challenges of 5G are categorized into the following two headings –

- Technological Challenges
- Common Challenges

Technological Challenges

- **Inter-cell Interference** – This is one of the major technological issues that need to be solved. There is variations in size of traditional macro cells and concurrent small cells that will lead to interference.



- **Efficient Medium Access Control** – In a situation, where dense deployment of access points and user terminals are required, the user throughput will be low, latency will be high, and hotspots will not be competent to cellular technology to provide high throughput. It needs to be researched properly to optimize the technology.
- **Traffic Management** – In comparison to the traditional human to human traffic in cellular networks, a great number of Machine to Machine (M2M) devices in a cell may cause serious system challenges i.e. radio access network (RAN) challenges, which will cause overload and congestion.

Common Challenges

- **Multiple Services** – Unlike other radio signal services, 5G would have a huge task to offer services to heterogeneous networks, technologies, and devices operating in different geographic regions. So, the challenge is of standardization to provide dynamic, universal, user-centric, and data-rich wireless services to fulfil the high expectation of people.



- **Infrastructure** – Researchers are facing technological challenges of standardization and application of 5G services.
- **Communication, Navigation, & Sensing** – These services largely depend upon the availability of radio spectrum, through which signals are transmitted. Though 5G technology has strong computational power to process the huge volume of data coming from different and distinct sources, but it needs larger infrastructure support.
- **Security and Privacy** – This is one of the most important challenges that 5G needs to ensure the protection of personal data. 5G will have to define the uncertainties related to security threats including trust, privacy, cybersecurity, which are growing across the globe.
- **Legislation of Cyberlaw** – Cybercrime and other fraud may also increase with the high speed and ubiquitous 5G technology. Therefore, legislation of the Cyberlaw is also an imperative issue, which largely is governmental and political (national as well as international issue) in nature.

5G - Future Scope

Several researches and discussions are going on across the world among technologists, researchers, academicians, vendors, operators, and governments about the innovations, implementation, viability, and security concerns of 5G.

As proposed, loaded with multiple advance features starting from the super high speed internet service to smooth ubiquitous service, 5G will unlock many of the problems. However, the question is — in a situation, where the previous technologies (4G and

3G) are still under process and in many parts yet to be started; what will be the future of 5G?



5th generation technology is designed to provide incredible and remarkable data capabilities, unhindered call volumes, and immeasurable data broadcast within the latest mobile operating system. Hence, it is more intelligent technology, which will interconnect the entire world without limits. Likewise, our world would have universal and uninterrupted access to information, communication, and entertainment that will open a new dimension to our lives and will change our life style meaningfully.

Moreover, governments and regulators can use this technology as an opportunity for the good governance and can create healthier environments, which will definitely encourage continuing investment in 5G, the next generation technology.